7.3 Probability

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS conditional probability • independence

Probability

Objective

Use Venn diagrams, tree diagrams and the addition/multiplication laws, with conditional probability and sampling without replacement (objectives 3.1–3.4).

1 Key knowledge and worked examples

Laws. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(A \cap B) = P(A)P(B \mid A)$; independent $\iff P(A \cap B) = P(A)P(B)$. Conditional reversal: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$. Without replacement, second-layer fractions drop by one in numerator and denominator. Fill the Venn diagram first; answers fall out.

Worked example – Venn (WST01 June 2025 Q4). $P(C \cup D) = 0.59$, $P(D) = 0.45$, $P(C \mid D) = 0.2$: $P(C \cap D) = 0.2 \times 0.45 = 0.09$ and $P(C) = 0.59 + 0.09 - 0.45 = 0.23$.

Worked example – tree and reversal (WST01 June 2025 Q7). 3 white, 6 yellow beads, two drawn without replacement: exactly one yellow $= \frac{6}{9}\frac{3}{8} + \frac{3}{9}\frac{6}{8} = \frac{1}{2}$; at least one white $= 1 - \frac{6}{9}\frac{5}{8} = \frac{7}{12}$; both white given at least one white $= \dfrac{\frac{3}{9}\frac{2}{8}}{\frac{7}{12}} = \dfrac{1}{7}$.

2 Practice

2.1 $P(A) = 0.6$, $P(B) = 0.5$, $P(A \cup B) = 0.85$. Find $P(A \cap B)$ and state whether A and B are independent. [3]

2.2 A bag has 5 red and 4 blue counters; two are drawn without replacement. Find the probability that both are the same colour. [3]

2.3 $P(X \cap Y) = 0.21$ and $P(X) = 0.35$. Find $P(Y \mid X)$ and $P(X' \cap Y)$ if Y and X are independent. [4]

3 Exam-style questions

3.1 *(Edexcel IAL WST01 June 2025 Q4.)* Two events C and D are such that $P(C \cup D) = 0.59$, $P(D) = 0.45$ and $P(C \mid D) = 0.2$. (a) Find $P(C \cap D)$. (b) Find $P(C)$. [4]

3.2 *(Edexcel IAL WST01 June 2025 Q7, condensed.)* Riley's bag contains 3 white beads and 6 yellow beads. Two beads are selected at random, one at a time, without replacement. (a) Complete a tree diagram for the selection. (b) Find the probability that exactly one bead is yellow. (c) Find the probability that at least one bead is white. (d) Given that at least one bead is white, find the probability that both beads are white. [9]

Solutions

2.1 $P(A \cap B) = 0.6 + 0.5 - 0.85 = 0.25$. Independent would need $P(A \cap B) = 0.6 \times 0.5 = 0.3 \neq 0.25$: NOT independent.

2.2 $P(\mathrm{RR}) + P(\mathrm{BB}) = \frac{5}{9}\frac{4}{8} + \frac{4}{9}\frac{3}{8} = \frac{20+12}{72} = \frac{32}{72} = \frac{4}{9}$.

2.3 $P(Y \mid X) = \frac{0.21}{0.35} = 0.6$. Independence $\Longrightarrow P(Y) = 0.6$; then $P(X' \cap Y) = 0.4 \times 0.6 = 0.24$.

3.1 (a) $P(C \mid D) = \frac{P(C \cap D)}{P(D)}$ gives $P(C \cap D) = 0.2 \times 0.45 = 0.09$. (b) $P(C) = P(C \cup D) + P(C \cap D) - P(D) = 0.59 + 0.09 - 0.45 = 0.23$.

3.2 (a) First layer: yellow $\frac{6}{9}$, white $\frac{3}{9}$; after yellow: yellow $\frac{5}{8}$, white $\frac{3}{8}$; after white: yellow $\frac{6}{8}$, white $\frac{2}{8}$. (b) $\frac{6}{9} \cdot \frac{3}{8} + \frac{3}{9} \cdot \frac{6}{8} = \frac{18+18}{72} = \frac{1}{2}$. (c) $1 - P(\text{no white}) = 1 - \frac{6}{9} \cdot \frac{5}{8} = 1 - \frac{5}{12} = \frac{7}{12}$. (d) $P(\text{both white} \mid \text{at least one white}) = \dfrac{\frac{3}{9} \cdot \frac{2}{8}}{\frac{7}{12}} = \dfrac{1/12}{7/12} = \dfrac{1}{7}$.

Local review guidance: multiplication along tree branches with consistent labels; "at least one" via the complement; conditional reversal dividing by the CONDITIONING event's probability. This is authored classroom guidance.