7.2 Measures of location and dispersion

Name: _____ Class: _____ Date: _____ **Total: 18 marks**

KEY WORDS variance • standard deviation

Measures of location and dispersion

Objective

Combine data sets through $\sum x$ and $\sum x^2$, use $\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$, and state the effect of coding on means, variances and standard deviations (objectives 2.2–2.3).

1 Key knowledge and worked examples

Combining and coding. For merged groups: $\sum x = n_1\bar{x}_1 + n_2\bar{x}_2$ (add the totals), $\sum x^2$ adds directly, then reapply $\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$ with the combined n and mean. Adding a constant shifts the mean, leaves variance and SD unchanged; multiplying scales the mean and the SD by the factor, the variance by its square.

Worked example – two classes merged (WST01 June 2025 Q2). A: $n = 24$, $\bar{x} = 47$, $\sum x^2 = 66876$; B: $n = 30$, $\bar{x} = b$, $\sum x^2 = 73826$; the 54 students average 45. $\sum x = 2430 \Rightarrow \sum x_B = 2430 - 24(47) = 1302 \Rightarrow b = 43.4$. Combined SD: $\sqrt{\frac{140702}{54} - 2025} = 24.1$. Adding 2 to each B mark: B's variance unchanged, combined mean up, combined SD DOWN (B moves closer to A).

2 Practice

2.1 A sample of 20 values has $\sum x = 340$ and $\sum x^2 = 6120$. Find the mean and standard deviation. [3]

2.2 Group P: $n = 15$, mean 12; group Q: $n = 10$, mean 18. Find the combined mean. [2]

2.3 Each value in a data set is doubled and then increased by 5. If the original variance was 9, find the new variance and state what happens to the standard deviation. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST01 June 2025 Q2.)* For class A ($n = 24$, mean 47, $\sum x^2 = 66876$) and class B ($n = 30$, mean b, $\sum x^2 = 73826$), the combined group of 54 students has mean mark 45. (a) Show that $b = 43.4$. (b) Find the standard deviation of the 54 marks. (c) Each class-B mark is increased by 2. State, with a reason, the effect on (i) the variance of the class-B marks, (ii) the mean of all 54 marks, (iii) the standard deviation of all 54 marks. [7]

3.2 *(Original.)* A data set has mean μ and standard deviation σ. Write down the mean and standard deviation after the coding $y = 4 - 3x$, and the variance of y. [3]

Solutions

2.1 Mean $= \frac{340}{20} = 17$; $\sigma^2 = \frac{6120}{20} - 17^2 = 306 - 289 = 17$, so $\sigma = 4.12$.

2.2 $\bar{x} = \frac{15(12)+10(18)}{25} = \frac{360}{25} = 14.4$.

2.3 New variance $= 2^2 \times 9 = 36$; the standard deviation doubles from 3 to 6 (the +5 shifts everything but changes no spread).

3.1 (a) $\sum x = 54(45) = 2430$; class A contributes $24 \times 47 = 1128$, so $\sum x_B = 1302$ and $b = \frac{1302}{30} = 43.4$. (b) Combined $\sum x^2 = 66876 + 73826 = 140702$, so $\sigma = \sqrt{\frac{140702}{54} - 45^2} = \sqrt{24.095\ldots} = 24.1$. (c)(i) No effect – a uniform shift changes no measure of spread. (ii) Increases – B's total rises by 60, so the combined total and mean rise ($\frac{2490}{54} = 46.1$). (iii) Decreases – the two class means (49 and 45.4) are now closer together, so the combined data is less spread out.

3.2 Mean $= 4 - 3\mu$; SD $= 3\sigma$; variance $= 9\sigma^2$ (the shift 4 adds nothing to spread; the factor 3 scales SD linearly and variance quadratically).

Local review guidance: totals formed with $n\bar{x}$ before any division; variance via $\frac{\sum x^2}{n} - \bar{x}^2$ with the COMBINED mean; coding effects stated with a spread-based reason. This is authored classroom guidance.