6.2 Equilibrium with elastic strings

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS modulus of elasticity • natural length

Equilibrium with elastic strings

Objective

Combine moments with Hooke's law for hinged bodies held by elastic strings, and resolve forces in elastic-string equilibrium (objectives 2.1, 5.2).

1 Key knowledge and worked examples

Geometry first. For a hinged rod held by an elastic string, draw the exact layout: the horizontal and vertical offsets of every attachment from the hinge give both the moment arms and the string direction. Then $T = \frac{\lambda e}{L}$ converts tension to extension, and the natural length falls out.

Worked example (WME03 June 2025 Q1). Uniform rod AB (mass m, length $2a$) hinged at A; an elastic string with $\lambda = 2mg$ and natural length L runs from B to C, where AC is horizontal and the string is vertical with $BC = 1.2a$ – B sits $1.6a$ horizontally from A and $1.2a$ below it (3–4–5), so the rod's midpoint is $0.8a$ horizontally from the vertical through A. Moments about A: $T(1.6a) = mg(0.8a)$, so $T = \frac{1}{2}mg$; then $\frac{1}{2}mg = \frac{2mg}{L}(1.2a - L)$ gives $L = 0.96a = \frac{24}{25}a$.

2 Practice

2.1 A light elastic string (natural length $2a$, modulus $3mg$) hangs vertically with a mass m in equilibrium. Find the extension in terms of a. [2]

2.2 A uniform rod AB of mass m and length $3a$ rests horizontally, held by a vertical elastic string at B ($\lambda = mg$, natural length a) and a smooth support at A. Find the string's extension. [3]

2.3 Two identical elastic strings (each λ, natural L) hold a mass m symmetrically, each making 30° with the vertical. Find the extension of each. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME03 June 2025 Q1.)* A uniform rod AB has mass m and length $2a$. The end A is freely hinged to a fixed point. A light elastic string of modulus of elasticity $2mg$ and natural length L has one end attached to B and the other to a fixed point C, where AC is horizontal. The rod rests in equilibrium with the string taut and vertical and $BC = 1.2a$. (a) Find the tension in the string in terms of m and g. (b) Find L in terms of a. [7]

3.2 *(Original.)* A uniform beam AB of mass $2m$ and length $4a$ is hinged at A and held at $45°$ above the horizontal by a light elastic string from B to a point C directly above A with $AC = 2\sqrt{2}\,a$, so that the string is horizontal ($\lambda = 4mg$, natural length $2.5a$). Show that the beam cannot rest in equilibrium in this position. [6]

Solutions

2.1 $mg = \frac{3mg\,e}{2a}$: $e = \frac{2a}{3}$.

2.2 Moments about A: $T(3a) = mg \cdot \frac{3a}{2}$, so $T = \frac{mg}{2}$; then $\frac{mg}{2} = \frac{mg\,e}{a}$ gives $e = \frac{a}{2}$.

2.3 Vertically: $2T\cos 30^\circ = mg$, so $T = \frac{mg}{\sqrt{3}}$; extension $e = \frac{mgL}{\sqrt{3}\,\lambda}$.

3.1 (a) The geometry: B is at horizontal distance $\sqrt{(2a)^2 - (1.2a)^2} = 1.6a$ from the vertical through A and $1.2a$ below the level of AC; the rod's midpoint is $0.8a$ horizontally from the vertical through A. Moments about A: $T \cdot 1.6a = mg \cdot 0.8a$, so $T = \frac{1}{2}mg$. (b) Hooke's law: $\frac{1}{2}mg = \frac{2mg}{L}(1.2a - L)$, i.e. $\frac{L}{4} = 1.2a - L$, so $\frac{5L}{4} = 1.2a$ and $L = 0.96a = \frac{24}{25}a$.

3.2 Moments about A: the beam's midpoint is at horizontal distance $2a\cos 45^\circ = \sqrt{2}a$ and the string at B acts horizontally at height $4a\sin 45^\circ = 2\sqrt{2}a$, so equilibrium needs $T \cdot 2\sqrt{2}a = 2mg \cdot \sqrt{2}a$, i.e. $T = mg$. But the string's length is fixed by the geometry: B and C are both at height $2\sqrt{2}a$, so the string has length $2\sqrt{2}a$ and extension $2\sqrt{2}a - 2.5a \approx 0.328a$, giving an available tension $T = \frac{4mg(0.328a)}{2.5a} \neq mg$. The tension the geometry supplies cannot match the tension the moments demand, so the beam cannot rest in this position.

Local review guidance: moment arms from the hinge with the string's perpendicular distance; Hooke's law with the correct natural length; geometric feasibility checked. This is authored classroom guidance.