5.6 Statics of rigid bodies

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS limiting equilibrium • moment

Statics of rigid bodies

Objective

Solve rigid-body equilibrium with moments about the unknown-killing pivot, resolve contact components, and close limiting-equilibrium problems into equations in $\tan\theta$ (objectives 5.1–5.2).

1 Key knowledge and worked examples

Rigid-body routine. Draw every force; take moments about the point with the most unknowns (a smooth contact contributes one unknown perpendicular force); resolve horizontally and vertically for the rest. Rough ground at limiting equilibrium: $F = \mu R$ horizontal, R vertical. To finish in $\tan\theta$: divide by $\cos^2\theta$ and use $\sec^2\theta = 1 + \tan^2\theta$.

Worked example – rod against a smooth rail (WME02 June 2025 Q6). Uniform rod AB (m, $2a$), A on rough ground, smooth rail at C with $AC = 1.5a$, inclination θ. Moments about A: $S(1.5a) = mga\cos\theta$, so $S = \frac{2mg\cos\theta}{3}$. Vertically: $V = mg - S\cos\theta = \frac{mg}{3}(3 - 2\cos^2\theta)$. With $\mu = \frac{4}{7}$ and $F = S\sin\theta$: $\frac{4}{7}\cdot\frac{mg}{3}(3 - 2\cos^2\theta) = \frac{2mg}{3}\sin\theta\cos\theta$, which becomes $6\tan^2\theta - 7\tan\theta + 2 = 0$.

2 Practice

2.1 A uniform beam of weight 200 N and length 6 m rests on two supports at its ends. A 500 N load hangs 2 m from the left support. Find both reactions. [3]

2.2 A non-uniform rod of weight 120 N rests in limiting equilibrium on rough horizontal ground against a smooth vertical wall, inclined at 40°; the ground reaction acts at the rod's lower end. Given the ground friction is 48 N, find μ. [3]

2.3 A ladder of mass 10 kg and length 5 m leans against a smooth vertical wall at 60° to the horizontal ground (rough). Find the wall's reaction by taking moments about the foot. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME02 June 2025 Q6.)* A uniform rod AB has mass m and length $2a$. The end A of the rod rests on rough horizontal ground. A horizontal rail is fixed to a wall at a height above the ground. The rod rests in equilibrium against the rail at the point C, where $AC = 1.5a$, with the vertical plane of the rod perpendicular to the rail. The rod is inclined at angle θ to the floor, and the magnitude of the normal reaction of the rail on the rod at C is S. (a) Show that $S = \frac{2mg\cos\theta}{3}$. (b) The vertical component of the force of the floor on the rod at A is V. Show that $V = \frac{mg}{3}(3 - 2\cos^2\theta)$. (c) The coefficient of friction between the rod and the floor is $\frac{4}{7}$, and the rod is in limiting equilibrium. Show that $6\tan^2\theta - 7\tan\theta + 2 = 0$. [12]

3.2 *(Original.)* A uniform ladder of mass 12 kg and length 6 m leans against a smooth vertical wall, with its foot on rough horizontal ground. The ladder is at 50° to the horizontal and in limiting equilibrium. Find the coefficient of friction between the ladder and the ground (3 s.f.). [5]

Solutions

2.1 Moments about the left support: $R_R(6) = 200(3) + 500(2) = 1600$, so $R_R = 266.7$ N; $R_L = 700 - 266.7 = 433.3$ N.

2.2 The ground reaction $R =$ total vertical force $= 120$ N (the smooth wall gives only a horizontal force). So $\mu = \frac{48}{120} = 0.4$.

2.3 Moments about the foot: $S_{\text{wall}}(5\sin 60°) = 10g(2.5\cos 60°)$, so $S_{\text{wall}} = \frac{10(9.8)(1.25)}{5(0.866)} = 28.3$ N.

3.1 (a) The rail is smooth, so its reaction S is perpendicular to the rod at C. Moments about A (eliminates the floor forces): the weight mg acts at the midpoint, distance a along the rod, with moment arm $a\cos\theta$; S acts at distance $1.5a$ with full moment arm: $S(1.5a) = mg\,a\cos\theta$, so $S = \frac{2mg\cos\theta}{3}$ ✓. (b) Vertical equilibrium: S is perpendicular to the rod, so at rod angle θ its vertical component is $S\cos\theta$ and $V = mg - \frac{2mg\cos\theta}{3}\cos\theta = mg - \frac{2mg}{3}\cos^2\theta = \frac{mg}{3}(3 - 2\cos^2\theta)$ ✓. (c) Horizontal equilibrium: $F = S\sin\theta$. Limiting: $F = \frac{4}{7}V$. So $\frac{4}{7}\cdot\frac{mg}{3}(3 - 2\cos^2\theta) = \frac{2mg}{3}\cos\theta\sin\theta$. Divide by $\frac{mg}{3}\cos^2\theta$: $\frac{4}{7}(3\sec^2\theta - 2) = 2\tan\theta$. With $\sec^2\theta = 1 + \tan^2\theta$: $\frac{4}{7}(1 + 3\tan^2\theta) = 2\tan\theta$, so $4 + 12\tan^2\theta = 14\tan\theta$, i.e. $6\tan^2\theta - 7\tan\theta + 2 = 0$ ✓.

3.2 Moments about the foot: $S(6\sin 50°) = 12g(3\cos 50°)$, so $S = \frac{12(9.8)(3)(0.6428)}{6(0.766)} = \frac{226.7}{4.596} = 49.3$ N. Horizontal: $F = S = 49.3$; vertical: $R = 12g = 117.6$ N. So $\mu = \frac{49.3}{117.6} = 0.419$ (0.419).

Local review guidance: moments about the pivot that eliminates the contact unknowns with correct moment arms (components of distances, not the distances); components of the perpendicular reaction resolved with the right angle; (DM1) the $\sec^2\theta$ substitution completing the $\tan\theta$ equation. This is authored classroom guidance.