

KEY WORDS coefficient of restitution • momentum

Objective

1 Key knowledge and worked examples

Worked example – the full algebra (WME02 June 2025 Q5a–c). P ($2m$, speed u) strikes Q (m , at rest): $2u = 2v_P + v_Q$ and $eu = v_Q - v_P$ give $v_Q = \frac{2(1+e)u}{3}$ and $v_P = \frac{(2-e)u}{3}$. KE after = $\frac{mu^2}{3}(2 + e^2)$, so the loss is $\frac{mu^2}{3}(1 - e^2)$ – the constant $k = \frac{1}{3}$. If the impulse on Q is $\frac{8mu}{9}$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9}$ gives $e = \frac{1}{3}$.

2 Practice

2.1 A 3 kg particle at 4 m s^{-1} catches a 1 kg particle at 1 m s^{-1} , both moving right; they coalesce. Find the common speed and the energy lost. [4]

2.2 Two equal masses collide with $e = \frac{1}{2}$: A at 6 m s^{-1} hits B at rest. Find both speeds after. [3]

2.3 For a direct collision, show that the total kinetic energy is conserved when $e = 1$ is impossible unless the relative speed is zero – one line from the loss formula $\text{loss} = \frac{m_A m_B}{2(m_A + m_B)}(1 - e^2)(u_A - u_B)^2$. [2]

3 Exam-style questions

3.1 (*Edexcel IAL WME02 June 2025 Q5a–c, condensed.*) Two particles P (mass $2m$) and Q (mass m) lie at rest on a smooth horizontal plane. P is projected with speed u directly towards Q, and they collide directly with coefficient of restitution e ($e > 0$). (a) Show that the speed of Q immediately after the collision is $\frac{2(1+e)u}{3}$. (b) Show that the total kinetic energy lost in the collision is $k(1 - e^2)mu^2$, where k is a constant to be determined. (c) Given that the magnitude of the impulse exerted on Q by P is $\frac{8}{9}mu$, find e . [12]

3.2 (*Original.*) A 2 kg particle moving right at 5 m s^{-1} collides directly with a 3 kg particle moving left at 1 m s^{-1} ; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]

Solutions

2.1 $3(4) + 1(1) = 4v$: $v = 3.25 \text{ m s}^{-1}$. KE before = $24 + 0.5 = 24.5 \text{ J}$; after = $\frac{1}{2}(4)(3.25)^2 = 21.125 \text{ J}$; lost = 3.375 J .

2.2 $6 = v_A + v_B$ and $\frac{1}{2}(6) = v_B - v_A$: $v_B = 4.5$, $v_A = 1.5 \text{ m s}^{-1}$ (A still moves forward, slower).

2.3 The loss is zero exactly when $(1 - e^2)(u_A - u_B)^2 = 0$: either $e = 1$ (perfectly elastic) or the particles were already moving together ($u_A = u_B$, no collision).

3.1 (a) Momentum: $2mu = 2mv_P + mv_Q$; restitution: $eu = v_Q - v_P$. Substituting $v_P = v_Q - eu$: $2u = 2v_Q - 2eu + v_Q = 3v_Q - 2eu$, so $v_Q = \frac{2u(1+e)}{3}$ ✓. (b) From (a), $v_P = \frac{2(1+e)u}{3} - eu = \frac{(2-e)u}{3}$. KE after = $\frac{1}{2}(2m) \left(\frac{(2-e)u}{3}\right)^2 + \frac{1}{2}m \left(\frac{2(1+e)u}{3}\right)^2 = \frac{mu^2}{9} [(2-e)^2 + 2(1+e)^2] = \frac{mu^2}{9} (6 + 3e^2) = \frac{mu^2}{3} (2 + e^2)$. Loss = $\frac{1}{2}(2m)u^2 - \frac{mu^2}{3}(2 + e^2) = mu^2 - \frac{2mu^2}{3} - \frac{e^2mu^2}{3} = \frac{1}{3}(1 - e^2)mu^2$, so $k = \frac{1}{3}$ ✓. (c) Impulse on Q = $m\Delta v$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9} \Rightarrow \frac{2(1+e)}{3} = \frac{8}{9} \Rightarrow 1 + e = \frac{4}{3}$, so $e = \frac{1}{3}$.

3.2 Right positive: momentum $2(5) + 3(-1) = 7 = 2v_A + 3v_B$; restitution $e = \frac{v_B - v_A}{5 - (-1)} = \frac{v_B - v_A}{6} = \frac{1}{4}$, so $v_B = v_A + 1.5$. Then $2v_A + 3v_A + 4.5 = 7$: $v_A = 0.5$, $v_B = 2 \text{ m s}^{-1}$ (both right). KE before = $25 + 1.5 = 26.5 \text{ J}$; after = $0.25 + 6 = 6.25 \text{ J}$; lost = 20.25 J .

Local review guidance: both equations with consistent signs; both speeds fully simplified before any energy statement; loss factorised to show the $(1 - e^2)$ factor. This is authored classroom guidance.