

Exercise walk-through

9.6

Mixed S3 practice

Pearson Edexcel · International A-Level · Further Mathematics

1. Unit S3 at a glance

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WST03/01: 1 hour 30 minutes, 75 marks; the booklet carries the Normal and chi-squared tables plus Spearman critical values. Must-know:

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2); \bar{X} \sim N(\mu, \sigma^2/n); s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1};$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}; r_s = 1 - \frac{6 \sum d^2}{n(n^2-1)}.$$

Check yourself

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Combine independent Normals (differences and absolute differences); four sampling methods with mechanical reasons; bias algebra and paired unbiased-estimator equations; recovering (μ, σ) from two sample-mean probabilities; z intervals and one/two-sample tests with the variance assumption; chi-squared for fit and contingency with df bookkeeping; Spearman's r_s and its table test.

Exercises 2.1·2.3

2.1 A sample of 50 has $\sum x = 350$ and $\sum x^2 = 2590$: find the unbiased estimates of μ and σ^2 . [3]

2.2 Independent $X \sim N(40, 2)$ and $Y \sim N(37, 3)$: find $P(X > Y)$. [3]

2.3 In a goodness-of-fit test with 9 cells and two parameters estimated, four tail cells are combined into one: state the final degrees of freedom. [2]

Exercises 2.1·2.3

2.1 A sample of 50 has $\sum x = 350$ and $\sum x^2 = 2590$: find the unbiased estimates of μ and σ^2 . [3]

2.1 $\bar{x} = 7$; $s^2 = \frac{2590 - 50(49)}{49} = \frac{140}{49} = 2.857$.

2.2 Independent $X \sim N(40, 2)$ and $Y \sim N(37, 3)$: find $P(X > Y)$. [3]

2.2 $X - Y \sim N(3, 5)$: $z = \frac{3}{\sqrt{5}} = 1.342$, $P = 0.9103$ (awrt 0.910).

2.3 In a goodness-of-fit test with 9 cells and two parameters estimated, four tail cells are combined into one: state the final degrees of freedom. [2]

2.3 9 cells $\rightarrow$ 6 after combining; $\text{df} = 6 - 1 - 2 = 3$.

Exercise 3.1

3.1 (*Edexcel IAL WST03 June 2025 Q8c.*) The weights of bags of sand are $S \sim N(18, 0.03^2)$ kg, bags of cement $C \sim N(25, 0.03^2)$ kg, and pallets $P \sim N(15, 0.2^2)$ kg, all independent. A load consists of 28 bags of sand, 5 bags of cement and a single pallet P_1 . Find the probability that the total weight of the load is less than $30P_1 + 190$ kg, showing the distribution of every combination used. [6]

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3.1 Let $R = \sum_{i=1}^{28} S_i + \sum_{i=1}^5 C_i - 29P_1$. Then $E(R) = 28(18) + 5(25) - 29(15) = 504 + 125 - 435 = 194$ and $\text{Var}(R) = 28(0.0009) + 5(0.0009) + 29^2(0.04) = 0.0252 + 0.0045 + 33.64 = 33.6697$, so $R \sim N(194, 33.6697)$. The event is $R < 190$: $z = \frac{190-194}{\sqrt{33.6697}} = \frac{-4}{5.804} = -0.689$, so $P = 1 - 0.7549 = 0.245$ (awrt 0.245).

Exercise 3.2

3.2 (*Original.*) A drinks machine's dispensing time is Normally distributed. A sample of 40 dispersions gives $\sum x = 92.0$ seconds and $\sum x^2 = 216.32$. (a) Find unbiased estimates of the mean and variance of the dispensing time. (b) Construct a 95% confidence interval for the mean time, justifying any substitution you make. (c) The manufacturer claims a mean of 2.2 seconds: test the claim at the 5% level (two-tailed) and conclude in context.^[11]

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3.2 (a) $\bar{x} = \frac{92.0}{40} = 2.3$ s; $s^2 = \frac{216.32 - 40(2.3)^2}{39} = \frac{216.32 - 211.6}{39} = \frac{4.72}{39} = 0.121$. (b) With $n = 40$ large, substitute s^2 for σ^2 : $SE = \sqrt{0.121/40} = 0.055$, so the interval is $2.3 \pm 1.96(0.055) = 2.3 \pm 0.108 = (2.192, 2.408)$ s. (c) $H_0: \mu = 2.2$ vs $H_1: \mu \neq 2.2$: $z = \frac{2.3 - 2.2}{0.055} = 1.82$ against two-tailed critical ± 1.96 . Since $1.82 < 1.96$, do not reject H_0 – insufficient evidence, at the 5% level, against the manufacturer's claimed mean of 2.2 seconds (the true mean may still differ; 2.2 is simply consistent with the sample).

Local review guidance: the inequality rearranged before any distribution is written; the pallet coefficient squared in the variance; large-sample substitution justified by n ; two-tailed comparison with ± 1.96 and a contextual conclusion. This is authored classroom guidance.