

Exercise walk-through

9.5

Rank correlation and combinations of normals

Pearson Edexcel · International A-Level · Further Mathematics

1. Unit S3 at a glance

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WST03/01: 1 hour 30 minutes, 75 marks; the booklet carries the Normal and chi-squared tables plus Spearman critical values. Must-know:

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2); \bar{X} \sim N(\mu, \sigma^2/n); s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1};$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}; r_s = 1 - \frac{6 \sum d^2}{n(n^2-1)}.$$

Check yourself

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Combine independent Normals (differences and absolute differences); four sampling methods with mechanical reasons; bias algebra and paired unbiased-estimator equations; recovering (μ, σ) from two sample-mean probabilities; z intervals and one/two-sample tests with the variance assumption; chi-squared for fit and contingency with df bookkeeping; Spearman's r_s and its table test.

Exercises 2.1·2.3

2.1 Two rankings of 8 items differ by $d = 1, 0, 2, 1, 1, 0, 1, 0$: find r_s . [2]

2.2 Independent $X \sim N(10, 4)$ and $Y \sim N(6, 9)$: write down the distribution of $2X - Y$. [3]

2.3 For those variables, find $P(X - Y > 6)$. [3]

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2.1 $\sum d^2 = 1 + 0 + 4 + 1 + 1 + 0 + 1 + 0 = 8$: $r_s = 1 - \frac{48}{8(63)} = 1 - 0.0952 = 0.905$ (3 dp).

2.2 Independent $X \sim N(10, 4)$ and $Y \sim N(6, 9)$: write down the distribution of $2X - Y$. [3]

2.2 $2X - Y \sim N(2(10) - 6, 4(4) + 9) = N(14, 25)$.

2.3 For those variables, find $P(X - Y > 6)$. [3]

2.3 $X - Y \sim N(4, 13)$: $z = \frac{6-4}{\sqrt{13}} = 0.555$, so $P = 1 - \Phi(0.555) = 1 - 0.7103 = 0.2897$.

Exercise 3.1

3.1 (*Edexcel IAL WST03 June 2025 Q2.*) At a talent show two judges ranked 10 performances (Judge 1: A–J as 1, 3, 5, 7, 9, 10, 2, 6, 8, 4; Judge 2: 2, 1, 3, 7, 9, 4, 5, 8, 10, 6). (a) Calculate Spearman's rank correlation coefficient. (b) Stating your hypotheses, test at the 1% level whether there is evidence of a positive correlation between the judges' rankings.

[7]

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3.1 (a) The rank differences (Judge 1 – Judge 2) are $-1, 2, 2, 0, 0, 6, -3, -2, -2, -2$, so $\sum d^2 = 1 + 4 + 4 + 0 + 0 + 36 + 9 + 4 + 4 + 4 = 66$ and $r_s = 1 - \frac{6(66)}{10(99)} = 1 - \frac{396}{990} = 0.6$. (b) $H_0: \rho = 0$, $H_1: \rho > 0$. The 1% one-tailed critical value for $n = 10$ is 0.7455. Since $0.6 < 0.7455$, do not reject H_0 – insufficient evidence of a positive correlation between the two judges' rankings.

Exercise 3.2

3.2 (*Edexcel IAL WST03 June 2025 Q8a–b, condensed.*) The weights of bags of sand are Normally distributed with mean 18 kg and standard deviation 0.03 kg, and bags of cement with mean 25 kg and standard deviation 0.03 kg (all independent). (a) Three bags of sand are selected at random: find the probability their total weight exceeds 54.1 kg. (b) Two bags of cement are selected: find the probability their weights differ by more than 0.02 kg.

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3.2 (a) $S_1 + S_2 + S_3 \sim N(54, 3(0.0009)) = N(54, 0.0027)$, standard deviation $\sqrt{0.0027} = 0.05196$. $z = \frac{54.1-54}{0.05196} = 1.924$, so $P = 1 - \Phi(1.924) = 1 - 0.9726 = 0.0274$. (b) $C_1 - C_2 \sim N(0, 2(0.0009)) = N(0, 0.0018)$, standard deviation $\sqrt{0.0018} = 0.04243$. By symmetry, $P(|C_1 - C_2| > 0.02) = 2 \left(1 - \Phi\left(\frac{0.02}{0.04243}\right)\right) = 2(1 - \Phi(0.4714)) = 2 \times 0.3192 = 0.638$.

Local review guidance: $\sum d^2$ shown term by term; critical value from the Spearman table with the correct tail; combined variances with squared coefficients; the doubling for an absolute difference. This is authored classroom guidance.