

Exercise walk-through

9.3

Confidence intervals and tests for means

Pearson Edexcel · International A-Level · Further Mathematics

1. Unit S3 at a glance

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WST03/01: 1 hour 30 minutes, 75 marks; the booklet carries the Normal and chi-squared tables plus Spearman critical values. Must-know:

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2); \bar{X} \sim N(\mu, \sigma^2/n); s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1};$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}; r_s = 1 - \frac{6 \sum d^2}{n(n^2-1)}.$$

Check yourself

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Combine independent Normals (differences and absolute differences); four sampling methods with mechanical reasons; bias algebra and paired unbiased-estimator equations; recovering (μ, σ) from two sample-mean probabilities; z intervals and one/two-sample tests with the variance assumption; chi-squared for fit and contingency with df bookkeeping; Spearman's r_s and its table test.

Exercises 2.1·2.3

2.1 A sample of 36 from a Normal population with $\sigma = 9$ gives $\bar{x} = 50$. Construct a 95% confidence interval for μ . [3]

2.2 For the same data, test $H_0: \mu = 52$ against $\mu < 52$ at the 5% level. [3]

2.3 State the two large-sample licences that let you use s in place of σ . [2]

Exercises 2.1·2.3

2.1 A sample of 36 from a Normal population with $\sigma = 9$ gives $\bar{x} = 50$. Construct a 95% confidence interval for μ . [3]

2.1 $50 \pm 1.96 \times \frac{9}{\sqrt{36}} = 50 \pm 1.96 \times 1.5 = (47.06, 52.94)$.

2.2 For the same data, test $H_0: \mu = 52$ against $\mu < 52$ at the 5% level. [3]

2.2 $z = \frac{50-52}{1.5} = -1.33$ vs critical -1.6449 : $-1.33 > -1.6449$, do not reject – insufficient evidence that $\mu < 52$.

2.3 State the two large-sample licences that let you use s in place of σ . [2]

2.3 The sample size is large enough that (i) $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ is approximately $N(0, 1)$, and (ii) by the Central Limit Theorem the population itself need not be Normal.

Exercise 3.1

3.1 (*Edexcel IAL WST03 June 2025 Q4.*) A biologist claims that the mean weight of grey squirrels is 200 g more than the mean weight of red squirrels. A research student believes the mean weight of grey squirrels is greater than 200 g more. Random samples of 80 grey and 80 red squirrels are taken; the weights of the grey squirrels are denoted by x grams with $\bar{x} = 500$ and $s_x^2 = 1156$, and the weights of the red squirrels by y grams with $\sum y = 23200$ and $\sum y^2 = 6741351$. (a) Find unbiased estimates of the mean and variance of the red squirrels' weights. (b) Using a suitable hypothesis test at the 5% level of significance, test the research student's belief, stating the critical value. (c) State the assumption you made about the variances.

[11]

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3.1 (a) $\bar{y} = \frac{23200}{80} = 290$; $s_y^2 = \frac{6741351 - 80(290)^2}{79} = \frac{13351}{79} = 169$. (b) $H_0: \mu_x - \mu_y = 200$, $H_1: \mu_x - \mu_y > 200$. $z = \frac{(500 - 290) - 200}{\sqrt{1156/80 + 169/80}} = \frac{10}{\sqrt{16.5625}} = 2.457$. One-tailed 5% critical value 1.6449. Since $2.457 > 1.6449$, reject H_0 – there is sufficient evidence, at the 5% level, that grey squirrels' mean weight is more than 200 g greater than red squirrels'. (c) The sample variances equal the corresponding population variances ($\sigma_x^2 = s_x^2$ and $\sigma_y^2 = s_y^2$).

Exercise 3.2

3.2 (*Original.*) A machine fills bottles; the fill volumes are Normally distributed with standard deviation 4 ml. A sample of 49 bottles gives $\bar{x} = 503$ ml. (a) Construct a 99% confidence interval for the mean fill. (b) The label claims 500 ml: test at the 1% level whether the mean fill exceeds 500 ml, and state the conclusion in context. [7]

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3.2 (a) $503 \pm 2.5758 \times \frac{4}{\sqrt{49}} = 503 \pm 1.47 = (501.53, 504.47)$ ml. (b) $H_0: \mu = 500$ vs $H_1: \mu > 500$: $z = \frac{503-500}{4/7} = 5.25$ vs critical 2.3263. Reject H_0 – strong evidence the mean fill exceeds 500 ml (the machine overfills on average).

Local review guidance: the null gap subtracted in the numerator; combined standard error with both variances over their own n ; critical value with the correct tail and the conclusion in context. This is authored classroom guidance.