

Exercise walk-through

9.2

Estimators, bias and the sample mean

Pearson Edexcel · International A-Level · Further Mathematics

1. Unit S3 at a glance

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WST03/01: 1 hour 30 minutes, 75 marks; the booklet carries the Normal and chi-squared tables plus Spearman critical values. Must-know:

$$aX \pm bY \sim N(a\mu_x \pm b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2); \bar{X} \sim N(\mu, \sigma^2/n); s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1};$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}; r_s = 1 - \frac{6 \sum d^2}{n(n^2-1)}.$$

Check yourself

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Combine independent Normals (differences and absolute differences); four sampling methods with mechanical reasons; bias algebra and paired unbiased-estimator equations; recovering (μ, σ) from two sample-mean probabilities; z intervals and one/two-sample tests with the variance assumption; chi-squared for fit and contingency with df bookkeeping; Spearman's r_s and its table test.

Exercises 2.1·2.3

2.1 $T = \frac{1}{3}(X_1 + X_2 + X_3)$ from a population with mean μ : show T is an unbiased estimator of μ . [2]

2.2 For a population with variance σ^2 , find $\text{Var}(T)$ in 2.1 and comment on why three observations are better than one. [2]

2.3 $\bar{X}$ is the mean of 64 observations from $N(25, 100)$. Find $P(\bar{X} > 26.75)$. [3]

Exercises 2.1·2.3

2.1 $T = \frac{1}{3}(X_1 + X_2 + X_3)$ from a population with mean μ : show T is an unbiased estimator of μ . [2]

2.1 $E(T) = \frac{1}{3}(\mu + \mu + \mu) = \mu$ – unbiased ✓.

2.2 For a population with variance σ^2 , find $\text{Var}(T)$ in 2.1 and comment on why three observations are better than one. [2]

2.2 $\text{Var}(T) = \frac{1}{9}(3\sigma^2) = \frac{\sigma^2}{3}$ – one third of a single observation's variance, so averaging three shrinks the standard deviation by a factor of $\sqrt{3}$.

2.3 $\bar{X}$ is the mean of 64 observations from $N(25, 100)$. Find $P(\bar{X} > 26.75)$. [3]

2.3 $\bar{X} \sim N(25, \frac{100}{64})$: $z = \frac{1.75}{1.25} = 1.4$, so $P = 1 - 0.9192 = 0.0808$.

Exercise 3.1

3.1 (*Edexcel IAL WST03 June 2025 Q6.*) A random sample $X_1, X_2, \dots, X_{20}$ is taken from a population with unknown mean μ and unknown variance σ^2 . (a) Explain why $\frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a statistic. (b) Show that $R = \frac{1}{2}X_1 + \frac{3}{4}X_{20}$ is a biased estimator of μ . (c) Write down the bias in terms of μ . (d) Given that $S = \frac{aX_1 - bX_2}{2}$ and $T = \frac{a\sum_{i=1}^{14} X_i + b\sum_{i=15}^{20} X_i}{20}$ are each unbiased estimators of μ , where a and b are constants, find the values of a and b . [8]

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3.1 (a) Its value is computed solely from sample observations – it involves no unknown population parameters. (b) $E(R) = \frac{1}{2}E(X_1) + \frac{3}{4}E(X_{20}) = (\frac{1}{2} + \frac{3}{4})\mu = \frac{5}{4}\mu \neq \mu$, so R is biased. (c) Bias $= \frac{5}{4}\mu - \mu = \frac{\mu}{4}$. (d) $E(S) = \frac{a-b}{2}\mu = \mu$ gives $a - b = 2$; $E(T) = \frac{14a+6b}{20}\mu = \mu$ gives $14a + 6b = 20$. Solving: $a = 1.6$, $b = -0.4$.

Exercise 3.2

3.2 (*Edexcel IAL WST03 June 2025 Q7.*) A random sample of n observations is taken from a normal distribution with mean μ and standard deviation σ ; $\bar{X}_n$ denotes the sample mean. Given that $P(\bar{X}_n < 45.2) = 0.3446$ and $P(\bar{X}_n > 75.2) = 0.0179$, (a) show that $\sigma = 12\sqrt{n}$; (b) find the value of μ ; (c) find $P(\bar{X}_n > 59)$. [11]

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3.2 (a) Standardising: $P(\bar{X}_n < 45.2) = 0.3446$ means the z-value is -0.4 , so $\mu - 0.4\frac{\sigma}{\sqrt{n}} = 45.2$; $P(\bar{X}_n > 75.2) = 0.0179$ means $z = 2.1$, so $\mu + 2.1\frac{\sigma}{\sqrt{n}} = 75.2$. Subtracting: $2.5\frac{\sigma}{\sqrt{n}} = 30$, so $\frac{\sigma}{\sqrt{n}} = 12$, i.e. $\sigma = 12\sqrt{n}$ ✓. (b) $\mu = 45.2 + 0.4(12) = 50$ (check: $75.2 - 2.1(12) = 50$ ✓). (c) SE = 12: $z = \frac{59-50}{12} = 0.75$, so $P = 1 - 0.7734 = 0.2266$.

Local review guidance: statistic defined by absence of parameters; bias in terms of the parameter; two z-equations from the table values before any algebra. This is authored classroom guidance.