

Exercise walk-through

6.4

Horizontal circular motion

Pearson Edexcel · International A-Level · Further Mathematics

3. Elastic strings

Elastic strings

$T = \frac{\lambda e}{L}$, energy = $\frac{\lambda e^2}{2L}$; a midpoint string is two half-strings with natural length halved.
 WME03 Q4: energy gain $\frac{1}{2}mga$ at $CD = \frac{3a}{2}$, and with resistance $\frac{1}{5}mg$ the launch
 $\sqrt{3ag/2}$ lands at $V = \sqrt{29ag/10}$. Q1's hinged rod: moments about A give $T = \frac{1}{2}mg$,
 then Hooke gives $L = \frac{24a}{25}$.

Check yourself

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$a = f(x)$ via $v dv/dx$ with constants closed; Hooke's law and elastic energy for whole and half strings; hinged-rod moments with the tension; SHM proof, period, amplitude, slow-time fractions; two-resolution horizontal circles; vertical-circle energy, $R = 0$, projectile continuation; hemisphere COM derivation; signed-mass composites; tilted equilibrium.

Exercises 2.1·2.3

2.1 A particle moves in a circle of radius 0.5 m at 2 revolutions per second. Find ω and the speed. [2]

2.2 A conical pendulum: string length l , angle 45° to the vertical, mass m . Find the angular speed in terms of g and l . [3]

2.3 For that pendulum, find the tension in terms of mg . [2]

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2.1 $\omega = 4\pi \text{ rad s}^{-1}$; $v = 0.5 \times 4\pi = 2\pi \text{ m s}^{-1}$.

2.2 A conical pendulum: string length l , angle 45° to the vertical, mass m . Find the angular speed in terms of g and l . [3]

2.2 Centrewards: $T \sin 45^\circ = m(l \sin 45^\circ)\omega^2 \implies T = ml\omega^2$. Vertically: $T \cos 45^\circ = mg \implies T = mg\sqrt{2}$. So $\omega^2 = \frac{g\sqrt{2}}{l}$, $\omega = \sqrt{\frac{g\sqrt{2}}{l}} = 2^{1/4}\sqrt{\frac{g}{l}}$.

2.3 For that pendulum, find the tension in terms of mg . [2]

2.3 $T = mg\sqrt{2}$.

Exercise 3.1

3.1 (*Edexcel IAL WME03 June 2025 Q2.*) One end of a light inextensible string of length a is attached to a fixed point O above a smooth horizontal table; the other end is attached to a particle P of mass m on the table. P moves on the table in a horizontal circle with constant angular speed $\sqrt{\frac{g}{2a}}$, the string taut and making a constant angle of 30° with the table. Find, in terms of m and g , the magnitude of the force exerted on P by the table. [7]

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3.1 Radius of the circle: $a \cos 30^\circ$. Newton's law towards the centre: $T \cos 30^\circ = m(a \cos 30^\circ)\omega^2 = ma \cos 30^\circ \cdot \frac{g}{2a}$, so $T = \frac{mg}{2}$. Vertically (no acceleration): $R + T \sin 30^\circ = mg$, so $R = mg - \frac{mg}{2} \cdot \frac{1}{2} = mg - \frac{mg}{4} = \frac{3mg}{4}$.

Exercise 3.2

3.2 (*Original.*) A car of mass m travels at speed v round a circular bend of radius r banked at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$ and there is no sideways frictional force on the car. (a) Show that $v^2 = \frac{3gr}{4}$. (b) Find the magnitude of the normal reaction in terms of m and g . [6]

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3.2 (a) No friction, so the only forces are the weight mg and the normal reaction R perpendicular to the banked surface. Resolving horizontally (centrewards) and vertically: $R \sin \theta = \frac{mv^2}{r}$ and $R \cos \theta = mg$. Dividing: $\tan \theta = \frac{v^2}{gr}$, so with $\tan \theta = \frac{3}{4}$, $v^2 = \frac{3gr}{4}$ ✓. (b) $R = \frac{mg}{\cos \theta}$ with $\cos \theta = \frac{4}{5}$ (3-4-5 triangle): $R = \frac{5}{4}mg$.

Local review guidance: the centreward resolution with the RADIUS (not the string length); vertical equilibrium including the normal reaction; answers in terms of m and g exactly. This is authored classroom guidance.