

Exercise walk-through

6.3

Simple harmonic motion

Pearson Edexcel · International A-Level · Further Mathematics

3. Elastic strings

Elastic strings

$T = \frac{\lambda e}{L}$, energy = $\frac{\lambda e^2}{2L}$; a midpoint string is two half-strings with natural length halved.
 WME03 Q4: energy gain $\frac{1}{2}mga$ at $CD = \frac{3a}{2}$, and with resistance $\frac{1}{5}mg$ the launch
 $\sqrt{3ag/2}$ lands at $V = \sqrt{29ag/10}$. Q1's hinged rod: moments about A give $T = \frac{1}{2}mg$,
 then Hooke gives $L = \frac{24a}{25}$.

Check yourself

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$a = f(x)$ via $v \, dv/dx$ with constants closed; Hooke's law and elastic energy for whole and half strings; hinged-rod moments with the tension; SHM proof, period, amplitude, slow-time fractions; two-resolution horizontal circles; vertical-circle energy, $R = 0$, projectile continuation; hemisphere COM derivation; signed-mass composites; tilted equilibrium.

Exercises 2.1·2.3

2.1 A particle on a smooth table is attached to a spring (λ , natural L) fixed at A, and rests at equilibrium at E. It is displaced a small distance and released: write down ω^2 and the period. [3]

2.2 For SHM with amplitude 0.4 m and $\omega = 3 \text{ rad s}^{-1}$, find the maximum speed and the speed when $x = 0.2 \text{ m}$. [3]

2.3 For that oscillation, find the time per oscillation for which $|x| \leq 0.2 \text{ m}$. [3]

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$$\mathbf{2.1} \quad \omega^2 = \frac{\lambda}{mL}; \text{ period } 2\pi\sqrt{\frac{mL}{\lambda}}.$$

2.2 For SHM with amplitude 0.4 m and $\omega = 3 \text{ rad s}^{-1}$, find the maximum speed and the speed when $x = 0.2 \text{ m}$. [3]

$$\mathbf{2.2} \quad v_{\max} = 0.4 \times 3 = 1.2 \text{ m s}^{-1}; v = 3\sqrt{0.4^2 - 0.2^2} = 3\sqrt{0.12} = 1.04 \text{ m s}^{-1} \text{ (3 sf).}$$

2.3 For that oscillation, find the time per oscillation for which $|x| \leq 0.2 \text{ m}$. [3]

2.3 $x = a \cos \omega t$, so $|x| \leq \frac{a}{2}$ means $|\cos \omega t| \leq \frac{1}{2}$: the phase sits in $[\frac{\pi}{3}, \frac{2\pi}{3}]$ modulo π – one third of each period: $\frac{1}{3} \times \frac{2\pi}{3} = \frac{2\pi}{9} \text{ s}$ (about 0.70 s).

Exercise 3.1

3.1 (*Edexcel IAL WME03 June 2025 Q7.*) A particle P of mass m lies at rest on a smooth horizontal table. One end of a light elastic string of natural length $3L$ and modulus of elasticity mg is attached to P and the other to a fixed point A on the table; a second string of natural length $2L$ and modulus $2mg$ runs from P to a fixed point B, with A, P, B collinear and $AB = 7L$. P rests in equilibrium at E with $AE = 4.5L$. P is held at C on AB with $AC = 5L$ and released from rest. (a) Show that P moves with simple harmonic motion about E and find the period. (b) Find the maximum speed of P. (c) Find the exact time, in any one oscillation, for which the speed of P is less than or equal to $\sqrt{gL/12}$. [16]

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3.1 (a) Equilibrium check at E: A-string extension $4.5L - 3L = 1.5L$, tension $\frac{mg(1.5L)}{3L} = \frac{mg}{2}$; B-string extension $2.5L - 2L = 0.5L$, tension $\frac{2mg(0.5L)}{2L} = \frac{mg}{2}$ ✓. At displacement x from E towards B: $m\ddot{x} = \frac{2mg}{2L}(\frac{L}{2} - x) - \frac{mg}{3L}(\frac{3L}{2} + x) = \frac{mg}{L}(\frac{L}{2} - x - \frac{L}{2} - \frac{x}{3}) = -\frac{4mg}{3L}x$. So P moves with SHM about E with $\omega^2 = \frac{4g}{3L}$, and the period is $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3L}{4g}} = \pi\sqrt{\frac{3L}{g}}$. (b) Released from C: amplitude $= 5L - 4.5L = \frac{L}{2}$; $v_{max} = \frac{L}{2}\sqrt{\frac{4g}{3L}} = \sqrt{\frac{gL}{3}}$. (c) The threshold $\sqrt{\frac{gL}{12}}$ is exactly half of v_{max} , so $|v| \leq \frac{v_{max}}{2} \iff |\cos \omega t| \leq \frac{1}{2} \iff$ the phase lies in $[\frac{\pi}{3}, \frac{2\pi}{3}]$ modulo π – one third of every period: $t = \frac{1}{3}\pi\sqrt{\frac{3L}{g}} = \pi\sqrt{\frac{L}{3g}}$.

Local review guidance: the $\ddot{x} = -\omega^2 x$ derivation with the equilibrium check visible; amplitude read from the release point; (dM1) time fractions via the sinusoid inequality, exact throughout. This is authored classroom guidance.