Exercise walk-through

5.4
Collisions and restitution

Pearson Edexcel · International A-Level · Further Mathematics

Exercises 2.1·2.3

2.1 A 3 kg particle at 4 $\mathrm{m\,s^{-1}}$ catches a 1 kg particle at 1 $\mathrm{m\,s^{-1}}$, both moving right; they coalesce. Find the common speed and the energy lost. [4]

2.2 Two equal masses collide with $e = \frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]

2.3 For a direct collision, show that the total kinetic energy is conserved when $e = 1$ is impossible unless the relative speed is zero – one line from the loss formula loss $= \frac{m_A m_B}{2(m_A+m_B)}(1-e^2)(u_A-u_B)^2$. [2]

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2.1 $3(4)+1(1) = 4v$: $v = 3.25\ \mathrm{m\,s^{-1}}$. KE before $= 24+0.5 = 24.5$ J; after $= \frac{1}{2}(4)(3.25)^2 = 21.125$ J; lost $= 3.375$ J.

2.2 Two equal masses collide with $e = \frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]

2.2 $6 = v_A + v_B$ and $\frac{1}{2}(6) = v_B - v_A$: $v_B = 4.5$, $v_A = 1.5\ \mathrm{m\,s^{-1}}$ (A still moves forward, slower).

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2.3 The loss is zero exactly when $(1-e^2)(u_A-u_B)^2 = 0$: either $e = 1$ (perfectly elastic) or the particles were already moving together ($u_A = u_B$, no collision).

Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q5a–c, condensed.)* Two particles P (mass $2m$) and Q (mass m) lie at rest on a smooth horizontal plane. P is projected with speed u directly towards Q, and they collide directly with coefficient of restitution e ($e > 0$). (a) Show that the speed of Q immediately after the collision is $\frac{2(1+e)u}{3}$. (b) Show that the total kinetic energy lost in the collision is $k(1 - e^2)mu^2$, where k is a constant to be determined. (c) Given that the magnitude of the impulse exerted on Q by P is $\frac{8}{9}mu$, find e. [12]

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3.1 (a) Momentum: $2mu = 2mv_P + mv_Q$; restitution: $eu = v_Q - v_P$. Substituting $v_P = v_Q - eu$: $2u = 2v_Q - 2eu + v_Q = 3v_Q - 2eu$, so $v_Q = \frac{2u(1+e)}{3}$ ✓. (b) From (a), $v_P = \frac{2(1+e)u}{3} - eu = \frac{(2-e)u}{3}$. KE after $= \frac{1}{2}(2m)\left(\frac{(2-e)u}{3}\right)^2 + \frac{1}{2}m\left(\frac{2(1+e)u}{3}\right)^2 = \frac{mu^2}{9}\left[(2-e)^2 + 2(1+e)^2\right] = \frac{mu^2}{9}(6+3e^2) = \frac{mu^2}{3}(2+e^2)$. Loss $= \frac{1}{2}(2m)u^2 - \frac{mu^2}{3}(2+e^2) = mu^2 - \frac{2mu^2}{3} - \frac{e^2mu^2}{3} = \frac{1}{3}(1-e^2)mu^2$, so $k = \frac{1}{3}$ ✓. (c) Impulse on Q $= m\Delta v$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9} \Rightarrow \frac{2(1+e)}{3} = \frac{8}{9} \Rightarrow 1 + e = \frac{4}{3}$, so $e = \frac{1}{3}$.

Exercise 3.2

3.2 *(Original.)* A 2 kg particle moving right at 5 $\mathrm{m\,s^{-1}}$ collides directly with a 3 kg particle moving left at 1 $\mathrm{m\,s^{-1}}$; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]

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3.2 Right positive: momentum $2(5) + 3(-1) = 7 = 2v_A + 3v_B$; restitution $e = \frac{v_B - v_A}{5-(-1)} = \frac{v_B - v_A}{6} = \frac{1}{4}$, so $v_B = v_A + 1.5$. Then $2v_A + 3v_A + 4.5 = 7$: $v_A = 0.5$, $v_B = 2\ \mathrm{m\,s^{-1}}$ (both right). KE before $= 25 + 1.5 = 26.5$ J; after $= 0.25 + 6 = 6.25$ J; lost $= 20.25$ J.

Local review guidance: both equations with consistent signs; both speeds fully simplified before any energy statement; loss factorised to show the $(1 - e^2)$ factor. This is authored classroom guidance.