Exercise walk-through

5.2
Centres of mass

Pearson Edexcel · International A-Level · Further Mathematics

Exercises 2.1·2.3

2.1 Particles of mass 2 kg, 3 kg and 5 kg sit at $x = 0$, 4 and 6 on the x-axis. Find their centre of mass. [2]

2.2 A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]

2.3 Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]

Exercises 2.1·2.3

2.1 Particles of mass 2 kg, 3 kg and 5 kg sit at $x = 0$, 4 and 6 on the x-axis. Find their centre of mass. [2]

2.1 $\bar{x} = \frac{2(0)+3(4)+5(6)}{10} = \frac{42}{10} = 4.2$ from the 2 kg particle.

2.2 A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]

2.2 Rectangle (area 24) at (3, 2); hole (area -4) at (1, 1) from the adjacent corner: $\bar{x} = \frac{24(3)-4(1)}{20} = \frac{68}{20} = 3.4$ m and $\bar{y} = \frac{24(2)-4(1)}{20} = \frac{44}{20} = 2.2$ m from the two outer edges at that corner.

2.3 Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]

2.3 The two 5-sides each have midpoint 1.5 m above the 6-side (height of the 5-5-6 triangle $= 4$); the 6-side has COM distance 0. Weights by length: $\bar{y} = \frac{5(1.5)+5(1.5)+6(0)}{16} = \frac{15}{16} = 0.9375$ m from the 6-side.

Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q3, condensed.)* A uniform rigid wire framework consists of two semicircular arcs AB and CD, both of centre O, joined by two straight pieces AC and DB. The arc AB has radius $2a$, the arc CD has radius a, $AC = DB = a$, and ACDB is a straight line. The arcs lie on opposite sides of ACDB. The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre. (a) Show that the centre of mass of the framework is $\frac{6a}{3\pi+2}$ from AB. (b) The point E lies on the arc AB with $\angle AOE = 90°$. The framework is freely suspended from A and hangs in equilibrium with AE at angle θ to the downward vertical. Show that $\tan\theta = \dfrac{3\pi - 1}{3\pi + 5}$. [10]

Exercise 3.1

3.1 *(Edexcel IAL WME02 June 2025 Q3, condensed.)* A uniform rigid wire framework consists of two semicircular arcs AB and CD, both of centre O, joined by two straight pieces AC and DB. The arc AB has radius $2a$, the arc CD has radius a, $AC = DB = a$, and ACDB is a straight line. The arcs lie on opposite sides of ACDB. The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre. (a) Show that the centre of mass of the framework is $\frac{6a}{3\pi+2}$ from AB. (b) The point E lies on the arc AB with $\angle AOE = 90°$. The framework is freely suspended from A and hangs in equilibrium with AE at angle θ to the downward vertical. Show that $\tan\theta = \dfrac{3\pi - 1}{3\pi + 5}$. [10]

3.1 (a) Table about the line AB: arc AB length $2\pi a$, COM $\frac{4a}{\pi}$ above; arc CD length πa, COM $\frac{2a}{\pi}$ BELOW; straights total $2a$ on the line: $2\pi a \cdot \frac{4a}{\pi} - \pi a \cdot \frac{2a}{\pi} + 2a \cdot 0 = 8a^2 - 2a^2 = 6a^2$; total length $= (3\pi+2)a$; so $\bar{x} = \frac{6a^2}{(3\pi+2)a} = \frac{6a}{3\pi+2}$ ✓ (the COM is on the AB side, distance d from the line). (b) By symmetry G lies on OE. In triangle AOE, $OA = OE = 2a$ and the angle at O is 90°, so $\angle OAE = 45°$; the COM is at distance d from A's vertical: in right triangle AOG, $\tan\angle OAG = \frac{d}{2a} = \frac{6}{2(3\pi+2)} = \frac{3}{3\pi+2}$. Hanging from A puts G on the vertical, so $\theta = \angle GAE = \angle OAE - \angle OAG = 45° - \angle OAG$, and $\tan\theta = \dfrac{1 - \frac{3}{3\pi+2}}{1 + \frac{3}{3\pi+2}} = \dfrac{3\pi - 1}{3\pi + 5}$ ✓.

Exercise 3.2

3.2 *(Original.)* A uniform semicircular lamina of radius 6 cm is freely suspended from one end P of its straight edge. Find the angle between the straight edge and the vertical, giving an exact tangent. [4]

Exercise 3.2

3.2 *(Original.)* A uniform semicircular lamina of radius 6 cm is freely suspended from one end P of its straight edge. Find the angle between the straight edge and the vertical, giving an exact tangent. [4]

3.2 The COM sits on the symmetry axis at $\frac{4r}{3\pi} = \frac{8}{\pi}$ cm from the centre O, i.e. $\frac{8}{\pi}$ from the straight edge, on the perpendicular through O. Suspended from P (one end of the edge, $OP = 6$), G hangs below P. In the right triangle with legs $OP = 6$ (along the edge) and the COM's perpendicular distance $\frac{8}{\pi}$ from the edge: the angle θ between the edge and the vertical satisfies $\tan\theta = \dfrac{8/\pi}{6} = \dfrac{4}{3\pi}$.

Local review guidance: pieces weighted by area/length with the correct booklet distance; opposite-side pieces signed; suspension angles via right-triangle trigonometry at the suspension point plus isosceles geometry. This is authored classroom guidance.