Exercise walk-through

4.1
Vectors in mechanics

Pearson Edexcel · International A-Level · Further Mathematics

Check yourself

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State modelling assumptions and their consequences; vector velocities, positions, bearings; multi-phase suvat and motion graphs; paired equations over a pulley with the 2T pulley force; impulse on one body and momentum conservation with rebounds; μR moving versus capped at rest; equilibrium by two resolutions; moments about successive tilting pivots.

Exercises 2.1·2.3

2.1 Find the magnitude of the velocity $3\mathbf{i} - 4\mathbf{j}$ m s^{-1} and the bearing on which the particle moves. [3]

2.2 Forces $(2\mathbf{i} + \mathbf{j})$ N and $(4\mathbf{i} - 3\mathbf{j})$ N act on a particle. Find the magnitude and direction of the resultant force. [3]

2.3 A particle has position vector $3\mathbf{i} - 2\mathbf{j}$ m when $t = 1$ and $12\mathbf{i} + 7\mathbf{j}$ m when $t = 4$. Find its velocity and its position vector at time t seconds. [4]

Exercises 2.1·2.3

2.1 Find the magnitude of the velocity $3\mathbf{i} - 4\mathbf{j}$ m s^{-1} and the bearing on which the particle moves. [3]

2.1 Magnitude $\sqrt{9+16} = 5$ m s^{-1}. The direction $(3, -4)$ is south-east; $\tan^{-1}(3/4) = 36.9°$ east of south, so the bearing is $180 - 37 = 143°$.

2.2 Forces $(2\mathbf{i} + \mathbf{j})$ N and $(4\mathbf{i} - 3\mathbf{j})$ N act on a particle. Find the magnitude and direction of the resultant force. [3]

2.2 Resultant $6\mathbf{i} - 2\mathbf{j}$: magnitude $\sqrt{36+4} = 2\sqrt{10} \approx 6.3$ N, direction $\tan^{-1}(2/6) = 18.4°$ south of east.

2.3 A particle has position vector $3\mathbf{i} - 2\mathbf{j}$ m when $t = 1$ and $12\mathbf{i} + 7\mathbf{j}$ m when $t = 4$. Find its velocity and its position vector at time t seconds. [4]

2.3 Velocity $= \dfrac{(12\mathbf{i} + 7\mathbf{j}) - (3\mathbf{i} - 2\mathbf{j})}{3} = 3\mathbf{i} + 3\mathbf{j}$; $\mathbf{r}(t) = (3\mathbf{i} - 2\mathbf{j}) + (t-1)(3\mathbf{i} + 3\mathbf{j}) = 3t\mathbf{i} + (3t - 5)\mathbf{j}$.

Exercises 2.4·3.1

2.4 Find the value of λ for which $\lambda\mathbf{i} + 2\mathbf{j}$ is perpendicular to $3\mathbf{i} - 4\mathbf{j}$. [3]

3.1 *(Edexcel IAL WME01 June 2025 Q2.)* At time t seconds, the position vector of particle A is $\mathbf{r}$ metres. When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$; when $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$. Particle B has position vector $\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$ metres. (a) Find the velocity of A. (b) Find $\mathbf{r}$ in terms of t. (c) Find the bearing of B from A when $t = 5$, giving your answer to the nearest degree. [7]

Exercises 2.4·3.1

2.4 Find the value of λ for which $\lambda\mathbf{i} + 2\mathbf{j}$ is perpendicular to $3\mathbf{i} - 4\mathbf{j}$. [3]

2.4 Perpendicular $\iff$ dot product zero: $3\lambda - 8 = 0$, so $\lambda = \frac{8}{3}$.

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3.1 (a) $\dfrac{(10\mathbf{i} + 4\mathbf{j}) - (-5\mathbf{i} + 16\mathbf{j})}{3} = \dfrac{15\mathbf{i} - 12\mathbf{j}}{3} = (5\mathbf{i} - 4\mathbf{j})\ \mathrm{m\,s^{-1}}$. (b) $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j}) + (t - 2)(5\mathbf{i} - 4\mathbf{j}) = (5t - 15)\mathbf{i} + (24 - 4t)\mathbf{j}$. (c) At $t = 5$: A is at $10\mathbf{i} + 4\mathbf{j}$; B is at $-5\mathbf{i} + 7\mathbf{j} + 5(2\mathbf{i} - 3\mathbf{j}) = 5\mathbf{i} - 8\mathbf{j}$; so $\overrightarrow{AB} = -5\mathbf{i} - 12\mathbf{j}$, magnitude 13, direction $\tan^{-1}(5/12) = 22.6°$ west of due south: bearing 203°.

Exercise 3.2

3.2 *(Original.)* A ship leaves the point with position vector $(2\mathbf{i} - \mathbf{j})$ km and moves with constant velocity $(3\mathbf{i} + 4\mathbf{j})$ $\mathrm{km\,h^{-1}}$, where $\mathbf{i}$ and $\mathbf{j}$ point east and north. (a) Find the bearing of the ship from the origin at the moment it leaves. (b) Find t when the ship is due east of the origin, and its distance from the origin then. [5]

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3.2 (a) Position $(2, -1)$: south-east quadrant, $\tan^{-1}(2/1) = 63.4°$ east of south, bearing $180 - 63 = 117°$. (b) Position $(2 + 3t)\mathbf{i} + (-1 + 4t)\mathbf{j}$: due east when the $\mathbf{j}$-component is zero, $t = 0.25$ h; position $2.75\mathbf{i}$, so the ship is 2.75 km due east of the origin.

Local review guidance: difference quotient with the correct time interval; position vectors generalised from one instant; bearings measured clockwise from north in the correct quadrant. This is authored classroom guidance.