

Exercise walk-through

# 2.5

## First and second order differential equations

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Pearson Edexcel · International A-Level · Further Mathematics

## Exercises 2.1·2.3

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**2.1** Solve  $\frac{dy}{dx} + 2y = e^{-x}$ . [4]

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**2.2** Solve  $x\frac{dy}{dx} + y = x^2$  for  $x > 0$ . [3]

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**2.3** Solve  $y'' + y = 0$  given  $y = 1$  and  $y' = 0$  at  $x = 0$ . [2]

## Exercises 2.1·2.3

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**2.1** Solve  $\frac{dy}{dx} + 2y = e^{-x}$ . [4]

**2.1** IF  $= e^{2x}$ :  $\frac{d}{dx}(ye^{2x}) = e^{2x}e^{-x} = e^x$ , so  $ye^{2x} = e^x + C$  and  $y = e^{-x} + Ce^{-2x}$ .

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**2.2** Solve  $x\frac{dy}{dx} + y = x^2$  for  $x > 0$ . [3]

**2.2** The left side is  $\frac{d}{dx}(xy)$ :  $xy = \frac{x^3}{3} + C$ , so  $y = \frac{x^2}{3} + \frac{C}{x}$ .

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**2.3** Solve  $y'' + y = 0$  given  $y = 1$  and  $y' = 0$  at  $x = 0$ . [2]

**2.3** Auxiliary  $m^2 + 1 = 0$ :  $y = A \cos x + B \sin x$ .  $y(0) = A = 1$ ;  $y' = -A \sin x + B \cos x$ ,  $y'(0) = B = 0$ . So  $y = \cos x$ .

## Exercises 2.4·3.1

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**2.4** Find the general solution of  $y'' - 4y = 12x$ .

[4]

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**3.1** (*Original.*) Find the general solution of  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$ .

[6]

## Exercises 2.4·3.1

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**2.4** Find the general solution of  $y'' - 4y = 12x$ . [4]

**2.4** Auxiliary  $m^2 - 4 = 0$ :  $m = \pm 2$ , CF =  $Ae^{2x} + Be^{-2x}$ . Trial PI =  $ax + b$ : substitution gives  $-4(ax + b) = 12x$ , so  $a = -3$ ,  $b = 0$ . General solution:  $y = Ae^{2x} + Be^{-2x} - 3x$ .

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**3.1** (*Original.*) Find the general solution of  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-2x}$ . [6]

**3.1** Auxiliary  $m^2 + 4m + 4 = (m + 2)^2 = 0$ : repeated root  $-2$ , so CF =  $(A + Bx)e^{-2x}$ . Since  $e^{-2x}$  and  $xe^{-2x}$  solve the homogeneous equation, try PI =  $\frac{\lambda x^2}{2}e^{-2x}$ : with  $y = \frac{\lambda x^2}{2}e^{-2x}$ ,  $y' = \lambda(x - x^2)e^{-2x}$  and  $y'' = \lambda(2x^2 - 4x + 1)e^{-2x}$ . Combining:  $y'' + 4y' + 4y = \lambda e^{-2x}[(2x^2 - 4x + 1) + 4(x - x^2) + 2x^2] = \lambda e^{-2x}$ . So  $\lambda = 1$  and the general solution is  $y = (A + Bx)e^{-2x} + \frac{x^2}{2}e^{-2x}$ .

## Exercise 3.2

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**3.2** (*Edexcel IAL WFM02 June 2025 Q4, condensed.*) Given that  $y = \lambda x e^{3x}$  is a particular integral of the differential equation  $4 \frac{d^2 y}{dx^2} - 11 \frac{dy}{dx} - 3y = 78e^{3x}$ , (a) determine the value of  $\lambda$ . (b) Hence determine the general solution of the differential equation. Given also that  $y = \frac{9}{2}$  and  $\frac{dy}{dx} = 0$  at  $x = 0$ , (c) determine the particular solution. [6]

## Exercise 3.2

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**3.2** (a)  $y' = \lambda(1 + 3x)e^{3x}$ ,  $y'' = \lambda(6 + 9x)e^{3x}$ , so  $4y'' - 11y' - 3y = \lambda e^{3x}(24 + 36x - 11 - 33x - 3x) = 13\lambda e^{3x} = 78e^{3x}$ , hence  $\lambda = 6$ . (b)  $4m^2 - 11m - 3 = (4m + 1)(m - 3)$ , so  $m = -\frac{1}{4}$  or  $3$ :  $y = Ae^{-x/4} + Be^{3x} + 6xe^{3x}$ . (c)  $y(0) = A + B = \frac{9}{2}$ ;  $y' = -\frac{A}{4}e^{-x/4} + 3Be^{3x} + 6(1 + 3x)e^{3x}$ , so  $y'(0) = -\frac{A}{4} + 3B + 6 = 0$ . From the first,  $A = \frac{9}{2} - B$ ; substituting:  $-\frac{1}{4}(\frac{9}{2} - B) + 3B = -6$ , so  $-\frac{9}{8} + \frac{13B}{4} = -6$ ,  $\frac{13B}{4} = -\frac{39}{8}$ ,  $B = -\frac{3}{2}$ , and  $A = 6$ . Particular solution:  $y = 6e^{-x/4} - \frac{3}{2}e^{3x} + 6xe^{3x}$ .

Local review guidance: correct IF or auxiliary treatment; trial PI adjusted for resonance when the plain trial dies; constants found from the full general solution, not the CF. This is authored classroom guidance.