

Exercise walk-through

## 2.4

# Loci, regions and $z \rightarrow w$ transformations

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Pearson Edexcel · International A-Level · Further Mathematics

## Exercises 2.1·2.3

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**2.1** Describe fully the locus given by  $|z - 2i| = 3$ . [2]

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**2.2** Describe fully the locus given by  $|z - 3| = |z + 1|$ . [2]

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**2.3** Show that the locus  $|z - 2| = 2|z + 1|$  is a circle, and state its centre and radius. [4]

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**2.1** Describe fully the locus given by  $|z - 2i| = 3$ . [2]

**2.1** Circle, centre  $2i$  (the point  $(0, 2)$ ), radius 3.

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**2.2** Describe fully the locus given by  $|z - 3| = |z + 1|$ . [2]

**2.2** Perpendicular bisector of the points 3 and  $-1$ : the vertical line  $\operatorname{Re} z = 1$ .

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**2.3** Show that the locus  $|z - 2| = 2|z + 1|$  is a circle, and state its centre and radius. [4]

**2.3** With  $z = x + iy$ :  $(x - 2)^2 + y^2 = 4[(x + 1)^2 + y^2]$ , so  $x^2 - 4x + 4 + y^2 = 4x^2 + 8x + 4 + 4y^2$ , i.e.  $3x^2 + 12x + 3y^2 = 0$ , so  $x^2 + 4x + y^2 = 0$ , i.e.  $(x + 2)^2 + y^2 = 4$ : circle, centre  $(-2, 0)$ , radius 2. (Check:  $z = 0$  gives  $|-2| = 2|1|$  ✓ and 0 lies on it.)

## Exercises 2.4·3.1

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**2.4** Describe the locus given by  $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ . [3]

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**3.1** (*Original.*) The transformation  $T$  from the  $z$ -plane to the  $w$ -plane is given by  $w = \frac{1}{z}$ ,  $z \neq 0$ . Show that  $T$  maps the circle  $|z-2| = 2$  onto the straight line  $\operatorname{Re} w = \frac{1}{4}$ , and sketch this line. [5]

## Exercises 2.4·3.1

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**2.4** Describe the locus given by  $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ . [3]

**2.4**  $\arg\left(\frac{z-1}{z+1}\right) = \arg(z-1) - \arg(z+1)$  is the angle at  $z$  subtended by the segment from  $-1$  to  $1$ . Equal to  $\frac{\pi}{2}$ : the semicircle  $x^2 + y^2 = 1$ ,  $y > 0$ , excluding the endpoints  $\pm 1$ .

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**3.1**  $w = \frac{1}{z} \Rightarrow z = \frac{1}{w}$ . Then  $|\frac{1}{w} - 2| = 2$  gives  $|1 - 2w| = 2|w|$ . Squaring:  $1 - 2w - 2\bar{w} + 4|w|^2 = 4|w|^2$ , so  $w + \bar{w} = \frac{1}{2}$ , i.e.  $\operatorname{Re} w = \frac{1}{4}$  – a vertical line in the  $w$ -plane,  $\frac{1}{4}$  unit to the right of the imaginary axis. (Check:  $z = 4$  is on the circle, and  $z = 4 \mapsto w = \frac{1}{4}$  ✓.)

## Exercise 3.2

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**3.2** (*Edexcel IAL WFM02 June 2025 Q3.*) The transformation  $T$  from the  $z$ -plane to the  $w$ -plane is given by  $w = \frac{9i - iz}{1 + z}$ ,  $z \neq -1$ . The point  $P$  represents the complex number  $z$ , where  $|z| = 1$ . (a) Sketch the locus of  $P$  as  $z$  varies. (b) Given that the image under  $T$  of the locus of  $P$ , where  $z \neq -1$ , is the line  $l$ , determine, in simplest form, a Cartesian equation for  $l$ . (c) Write down the image under  $T$  of the point  $z = i$  and verify that it lies on  $l$ . [9]

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**3.2** (a) Unit circle, centre the origin. (b)  $z = \frac{9i - w}{w + i}$ , so  $|9i - w| = |w + i| \cdot |z| = |w + i|$ . With  $w = u + iv$ :  $9i - w = -u + i(9 - v)$ , so  $u^2 + (9 - v)^2 = u^2 + (v + 1)^2$ , giving  $81 - 18v = 2v + 1$ , i.e.  $20v = 80$ :  $l$  is  $v = 4$ . (c)  $z = i$ :  $w = \frac{9i - i^2}{1 + i} = \frac{1 + 9i}{1 + i} = \frac{(1 + 9i)(1 - i)}{2} = \frac{10 + 8i}{2} = 5 + 4i$ , whose imaginary part is 4, so it lies on  $l$  ✓.

Local review guidance:  $z$  made the subject before any modulus work; modulus equation expanded in  $u, v$ ; line/circle description with geometry named (bisector, centre, radius). This is authored classroom guidance.