

Exercise walk-through

10.6

Mixed D1 practice

Pearson Edexcel · International A-Level · Further Mathematics

7. Critical path analysis

Critical paths

Precedence table to activity-on-arc network; dummies carry dependencies. Forward pass takes maxima (earliest event times), backward pass minima (latest). Float = latest finish – earliest start – duration; critical activities float zero. The example A(3), B(4), C(4), D(1), E(5) has critical path A–C–E = 12 days and D floats 2; Gantt bars run from earliest starts.

Check yourself

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Bubble and quick sorts pass by pass; first-fit, decreasing and full-bin placements; Kruskal and Prim (matrix form too); Dijkstra with permanence order and route recovery; odd-node pairings for the postman; TSP bounds both sides with nearest neighbour; dummies, forward and backward passes, floats, Gantt charts; LP formulation and vertex solutions with integer checks.

Exercises 2.1·2.3

2.1 Quick sort 5, 9, 2, 6, stating each pivot and the list after each partition. [3]

2.2 A network has edges RS 4, ST 1, RT 3, SU 5, TU 2. Find the MST by Prim from R. [3]

2.3 For the constraints $2x + y \leq 10$, $x + y \leq 8$, $x, y \geq 0$: maximise $x + 2y$ by the vertex method. [4]

Exercises 2.1·2.3

2.1 Quick sort 5, 9, 2, 6, stating each pivot and the list after each partition. [3]

2.1 Pivot (position 2 of 3) = 9: less = 5, 2, 6; greater = none $\rightarrow$ 5, 2, 6, **9**. Sublist 5, 2, 6 (pivot 2): less none; greater = 5, 6 $\rightarrow$ **2**, 5, 6. Sublist 5, 6 (pivot 5): less none; greater = 6 $\rightarrow$ **5**, 6. Final: 2, 5, 6, 9.

2.2 A network has edges RS 4, ST 1, RT 3, SU 5, TU 2. Find the MST by Prim from R. [3]

2.2 From R: edges RS 4, RT 3 – take RT(3). From {R,T}: edges to new vertices are RS 4, TS 1, TU 2 – take TS(1). From {R,T,S}: edges to U are SU 5 and TU 2 – take TU(2). MST = RT + TS + TU, total 6.

2.3 For the constraints $2x + y \leq 10$, $x + y \leq 8$, $x, y \geq 0$: maximise $x + 2y$ by the vertex method. [4]

2.3 Vertices: (0,0): 0; (5,0) ($2x + y = 10$): 5; (2,6) (intersection: $2x + y = 10$, $x + y = 8 \rightarrow x = 2, y = 6$): $2 + 12 = 14$; (0,8): 16. Maximum 16 at (0,8).

Exercise 3.1

3.1 (*Original.*) A rescue team must inspect every road between villages P, Q, R, S, T starting and ending at P. The roads and their lengths (km) are PQ 4, QR 2, RS 3, ST 5, PR 6, PS 2, QT 4, RT 1. (a) Find the total of all the road lengths. (b) Identify the odd-degree villages. (c) Find the least-weight pairing of the odd villages, listing every pairing. (d) Find the length of the shortest inspection route. [10]

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3.1 (a) Total = $4 + 2 + 3 + 5 + 6 + 2 + 4 + 1 = 27$. (b) Degrees: P 3 (PQ, PR, PS), Q 3 (PQ, QR, QT), R 4 (QR, RS, PR, RT), S 3 (RS, ST, PS), T 3 (ST, QT, RT): odd villages P, Q, S, T. (c) Pairings of {P, Q, S, T} (weights are shortest paths where no direct edge exists): $PQ + ST = 4 + 5 = 9$; $PS + QT = 2 + 4 = 6$; $PT + QS = 7 + 5 = 12$ (PT via PS–ST or PR–RT, both 7; QS via QR–RS, 5). Least = $PS + QT = 6$. (d) Route length = $27 + 6 = 33$.

Exercise 3.2

3.2 (*Original.*) A project has activities A (2), B (5), C (3 after A), D (4 after B), E (2 after C and D), F (1 after A). (a) Find the project duration and the critical path. (b) Find the total float of F. (c) The network is redrawn as a linear program limiting "workers on day 1" – the LP is maximise $W = x + y$ with x

le3, y

le2, $x + y$

le4: solve it and state the integer optimum.

[9]

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[9]

3.2 (a) Forward pass: A 0–2, F 0–3 (F after A: starts 2, ends 3), B 0–5, C starts 2 ends 5, D starts 5 ends 9, E starts 9 ends 11. Duration 11; critical path B–D–E ($5 + 4 + 2 = 11$). (b) F's latest finish = latest start of nothing downstream (F is an end activity): latest finish 11, earliest start 2, duration 1: float = $11 - 2 - 1 = 8$ days. (c) Vertices of $\{x \leq 3, y \leq 2, x + y \leq 4\}$: (0, 0), (3, 0), (2, 2), (0, 2): $W = 0, 3, 4, 2$: optimum $W = 4$ at (2, 2) – already integral ✓.

Local review guidance: pairing weights use shortest paths when the direct edge is absent; end activities' latest finish is the project duration; integer optima confirmed at neighbouring lattice points. This is authored classroom guidance.