

Exercise walk-through

10.4

Critical path analysis

Pearson Edexcel · International A-Level · Further Mathematics

7. Critical path analysis

Critical paths

Precedence table to activity-on-arc network; dummies carry dependencies. Forward pass takes maxima (earliest event times), backward pass minima (latest). Float = latest finish – earliest start – duration; critical activities float zero. The example A(3), B(4), C(4), D(1), E(5) has critical path A–C–E = 12 days and D floats 2; Gantt bars run from earliest starts.

Check yourself

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Bubble and quick sorts pass by pass; first-fit, decreasing and full-bin placements; Kruskal and Prim (matrix form too); Dijkstra with permanence order and route recovery; odd-node pairings for the postman; TSP bounds both sides with nearest neighbour; dummies, forward and backward passes, floats, Gantt charts; LP formulation and vertex solutions with integer checks.

Exercises 2.1·2.3

2.1 Activities A (2), B (5), C (3 after A), D (1 after A and B). Draw the precedence table as an activity network description (list arcs into and out of each event) and find the project duration. [4]

2.2 For a project with earliest event times 0, 4, 7, 9 and latest times 0, 4, 7, 9 along one chain, state the float of each activity on the chain. [2]

2.3 An activity lasts 6 days, its earliest start is day 5 and its latest finish is day 16. Find its total float. [2]

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2.1 Events: start s; A: $s \rightarrow e_1$ (2), B: $s \rightarrow e_2$ (5); C: $e_1 \rightarrow e_3$ (3); D: $e_1 \rightarrow e_3$ and $e_2 \rightarrow e_3$ – D depends on both A and B, so e_1 must connect to e_3 and e_2 to e_3 (a dummy from e_2 to e_1 if drawn with D leaving a shared node). Durations: $A-C = 2 + 3 = 5$; $B = 5$; D needs both done: D's earliest start = $\max(2, 5) = 5$. Project duration = $\max(5, 5 + 1) = 6$ days.

2.2 For a project with earliest event times 0, 4, 7, 9 and latest times 0, 4, 7, 9 along one chain, state the float of each activity on the chain. [2]

2.2 Earliest = latest at every event: every activity on the chain is critical, float 0.

2.3 An activity lasts 6 days, its earliest start is day 5 and its latest finish is day 16. Find its total float. [2]

2.3 Float = $16 - 5 - 6 = 5$ days.

Exercise 3.1

3.1 (*Original.*) A project has activities A (4), B (3), C (6 after A), D (2 after A and B), E (5 after C and D), F (3 after D). (a) Draw an activity network for the project (from the precedence table), using a dummy where necessary, and describe where the dummy is needed. (b) Carry out a forward pass and a backward pass, listing the earliest and latest times for each event. (c) Find the critical path and the duration of the project. (d) Find the total float of F.

[11]

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3.1 (a) Arcs: A from start to event 2; B from start to event 3; C from event 2 to event 4. D depends on A and B, so a dummy (duration 0) runs from event 2 to event 3 and D leaves event 3 for event 4. E and F both follow C and D (event 4); since two parallel arcs cannot share both endpoints, E runs from event 4 to event 5 and F from event 4 to event 6, with a dummy from event 5 to event 6 giving the single finish. (b) Forward: event 1 (start) 0; event 2 = 4 (after A); event 3 = $\max(4 + 0, 3) = 4$ (B finishes at 3, dummy carries 4); event 4 = $\max(4 + 6, 4 + 2) = 10$; finish = $\max(10 + 5, 10 + 3) = 15$. Backward: finish 15; event 4 = $\min(15 - 5, 15 - 3) = 10$; event 3 = $10 - 2 = 8$; event 2 = $\min(8 - 0, 10 - 6) = 4$; start = $\min(4 - 4, 8 - 3) = 0$ ✓. (c) Critical events: start 0/0, event 2 4/4, event 4 10/10, finish 15/15: critical path A–C–E, duration 15 days. (d) Float of F = latest finish – earliest start – duration = $15 - 10 - 3 = 2$ days.

Exercise 3.2

3.2 (*Original.*) Using the project of 3.1: (a) draw a Gantt chart for the project, labelling the critical activities; (b) state the minimum number of workers needed to avoid delaying any critical activity; (c) explain why F can start as late as day 12. [6]

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3.2 (a) Bars: A from 0 for 4; B from 0 for 3; C from 4 for 6; D from 4 for 2; E from 10 for 5; F from 10 for 3 with 2 days slack after it. Critical activities A, C, E on one unbroken chain. (b) One worker suffices for the critical chain if nothing else overlaps it in time – but B (0–3) and D (4–6) overlap A (0–4) and C (4–10): at least 2 workers are needed. (c) F's latest finish is 15 and it lasts 3 days, so it can start as late as $15 - 3 = 12$ (its earliest start is 10, giving the 2-day float).

Local review guidance: dummies carry exactly the dependency that shared endpoints cannot; forward pass takes maxima, backward pass minima; float from latest finish – earliest start – duration; Gantt bars from earliest starts with slack trailing. This is authored classroom guidance.