

Exercise walk-through

1.5

Parabola and rectangular hyperbola

Pearson Edexcel · International A-Level · Further Mathematics

Exercises 2.1·2.3

2.1 The parabola C has equation $y^2 = 8x$. Write down the coordinates of its focus and the equation of its directrix, and the parametric coordinates of a general point. [3]

2.2 For $y^2 = 20x$ at the point with $t = 3$, find the gradient of the tangent and its equation. [3]

2.3 The rectangular hyperbola $xy = 16$ has a point Q with $x = 8$. Find $\frac{dy}{dx}$ at Q and the equation of the normal at Q . [4]

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2.1 The parabola C has equation $y^2 = 8x$. Write down the coordinates of its focus and the equation of its directrix, and the parametric coordinates of a general point. [3]

2.1 $4a = 8 \Rightarrow a = 2$: focus $(2, 0)$; directrix $x = -2$; general point $(2t^2, 4t)$.

2.2 For $y^2 = 20x$ at the point with $t = 3$, find the gradient of the tangent and its equation. [3]

2.2 Point $(45, 30)$; gradient $\frac{1}{t} = \frac{1}{3}$. Tangent: $y - 30 = \frac{1}{3}(x - 45) \Rightarrow x - 3y + 45 = 0$.

2.3 The rectangular hyperbola $xy = 16$ has a point Q with $x = 8$. Find $\frac{dy}{dx}$ at Q and the equation of the normal at Q . [4]

2.3 $Q = (8, 2)$: $\frac{dy}{dx} = -\frac{16}{x^2} = -\frac{1}{4}$, so the normal has gradient 4: $y - 2 = 4(x - 8) \Rightarrow y = 4x - 30$.

Exercises 2.4·3.1

2.4 A point P on the parabola $y^2 = 4ax$ is 7 units from its focus. Find the distance from P to the directrix and the x -coordinate of P . [3]

3.1 (*Adapted from Edexcel IAL WFM01 June 2025 Q4.*) The rectangular hyperbola H has equation $xy = 36$; the point $P(6t, \frac{6}{t})$, $t \neq 0$, lies on H . (a) Use calculus to show that the normal to H at P has equation $ty = t^3x + 6(1 - t^4)$. (b) The normal meets the y -axis at A ; find the exact coordinates of A in terms of t . (c) Given $t = \frac{1}{2}$, find the exact area of triangle OPA . [8]

Exercises 2.4·3.1

2.4 A point P on the parabola $y^2 = 4ax$ is 7 units from its focus. Find the distance from P to the directrix and the x -coordinate of P . [3]

2.4 By the locus definition the distance to the directrix is also 7. Since both distances are horizontal, $PF = PM = x_P + a = 7$, so $x_P = 7 - a$.

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3.1 (a) $\frac{dy}{dx} = -\frac{36}{x^2}$; at $x = 6t$: $-\frac{36}{36t^2} = -\frac{1}{t^2}$; normal gradient t^2 : $y - \frac{6}{t} = t^2(x - 6t) \Rightarrow ty = t^3x + 6(1 - t^4)$ ✓. (b) $A = (0, \frac{6(1-t^4)}{t})$. (c) $t = \frac{1}{2}$: $P = (3, 12)$ and $A = (0, \frac{6(1-1/16)}{1/2}) = (0, 6 \cdot \frac{15}{16} \cdot 2) = (0, \frac{45}{4})$. Area $= \frac{1}{2} \times 3 \times \frac{45}{4} = \frac{135}{8}$.

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WFM01 June 2025 Q9.*) Figure: a parabola with focus F , directrix d , and a point P on the curve. The points Q and R lie on d with $PQ = PR = 12$, and P is 6 units from F . Determine the exact length QR . [4]

Exercise 3.2

3.2 (*Adapted from Edexcel IAL WFM01 June 2025 Q9.*) Figure: a parabola with focus F , directrix d , and a point P on the curve. The points Q and R lie on d with $PQ = PR = 12$, and P is 6 units from F . Determine the exact length QR . [4]

$$\mathbf{3.2} \quad PF = PM = 6; \quad QM = \sqrt{144 - 36} = \sqrt{108} = 6\sqrt{3}; \quad QR = 2 \times 6\sqrt{3} = 12\sqrt{3}.$$

Local review guidance: gradient from Cartesian differentiation, not parametric rules; exact coordinates and areas; focus-directrix arguments phrased through the locus definition. This is authored classroom guidance.