9.6 Mixed S3 practice

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS Central Limit Theorem

Mixed S3 practice

Objective

Combine the unit's tools on fuller problems: two-sample tests with variance recovery, estimator construction, chi-squared with parameter estimation, and pallet-type linear combinations (objectives 1.1, 3.1–3.8, 4.1–4.2).

1 Key knowledge and worked examples

Recurring chains. $\sum y$ and $\sum y^2 \to$ unbiased $\bar{y}$, $s_y^2 \to$ two-sample z. Litter counts $\to \hat{p} \to$ expected frequencies $\to$ X^2 with df lost to $\hat{p}$. A pallet problem rearranges the inequality until one Normal variable remains.

Worked example (WST03 June 2025 Q8c). With $S \sim N(18, 0.03^2)$, $C \sim N(25, 0.03^2)$, $P \sim N(15, 0.2^2)$: $T = \sum_1^{28} S_i + \sum_1^5 C_i + P_1$ and the event $T < 30P_1 + 190$ becomes $R = \sum S_i + \sum C_i - 29P_1 < 190$ where $R \sim N(194, 33.6697)$ – the pallet's coefficient 29 squares to 841 times its variance. Answer $\Phi\left(\frac{190-194}{5.804}\right) = 0.245$.

2 Practice

2.1 A sample of 50 has $\sum x = 350$ and $\sum x^2 = 2590$: find the unbiased estimates of μ and σ^2. [3]

2.2 Independent $X \sim N(40, 2)$ and $Y \sim N(37, 3)$: find $P(X > Y)$. [3]

2.3 In a goodness-of-fit test with 9 cells and two parameters estimated, four tail cells are combined into one: state the final degrees of freedom. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WST03 June 2025 Q8c.)* The weights of bags of sand are $S \sim N(18, 0.03^2)$ kg, bags of cement $C \sim N(25, 0.03^2)$ kg, and pallets $P \sim N(15, 0.2^2)$ kg, all independent. A load consists of 28 bags of sand, 5 bags of cement and a single pallet P_1. Find the probability that the total weight of the load is less than $30P_1 + 190$ kg, showing the distribution of every combination used. [6]

3.2 *(Original.)* A drinks machine's dispensing time is Normally distributed. A sample of 40 dispersions gives $\sum x = 92.0$ seconds and $\sum x^2 = 216.32$. (a) Find unbiased estimates of the mean and variance of the dispensing time. (b) Construct a 95% confidence interval for the mean time, justifying any substitution you make. (c) The manufacturer claims a mean of 2.2 seconds: test the claim at the 5% level (two-tailed) and conclude in context. [11]