8.6 Normal approximations and thresholds

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS continuity correction • normal approximation

Normal approximations and thresholds

Objective

Approximate binomial (and Poisson) distributions with the Normal distribution, apply the continuity correction, solve threshold problems for r and t, and combine with testing (objectives 1.3, 3.2, 4.6).

1 Key knowledge and worked examples

Approximation. For large n: $B(n,p) \approx N(np,\ np(1-p))$ (Poisson when instead p is small). "At least t" uses $t-0.5$; "at most r" uses $r+0.5$. Threshold searches: accumulate exact binomial probabilities for small n; standardise with the correction for large n and invert $z = \frac{x-0.5-\mu}{\sigma}$.

Worked example – thresholds both ways (WST02 June 2025 Q7c–d). $X \sim B(40, 0.3)$: largest r with $P(X < r) < 0.05$ (fewer than r): $P(X \leq 6) = 0.0238 < 0.05$ but $P(X \leq 7) = 0.0553 > 0.05$, so $r = 7$. For 200 customers: $B(200, 0.3) \approx N(60, 42)$; $P(X \geq t) = 0.9474$ means $\frac{t-0.5-60}{\sqrt{42}} = -1.62$, so $t = 0.5 + 60 - 1.62\sqrt{42} = 50.001$: $t = 50$.

2 Practice

2.1 $X \sim B(120, 0.4)$. State the approximating Normal distribution. [2]

2.2 Using that approximation, find $P(X > 55)$, with continuity correction. [3]

2.3 A Poisson variable has mean 100. State its Normal approximation and estimate $P(X < 90)$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q7, condensed.)* 30% of a random sample of 40 customers buying glasses also buy insurance; X is the number buying insurance. (a) State the distribution of X and one necessary assumption. (b) Find the largest r for which $P(X < r) < 0.05$ (fewer than r). (c) For a second sample of 200 customers, using a suitable Normal approximation, find the value of t such that $P(\text{at least } t \text{ of the 200 buy insurance}) = 0.9474$ (4 dp), showing all working. [9]

3.2 *(Original.)* A telephone line receives calls at an average rate of 4 per minute. Using a Normal approximation, estimate the probability that fewer than 200 calls arrive in one hour. [5]