8.4 Populations, samples and sampling distributions

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS sampling distribution • statistic

Populations, samples and sampling distributions

Objective

Distinguish census and sample with advantages, recognise statistics, and build sampling distributions by enumeration, wrapping counts into binomials for follow-ups (objectives 4.1–4.2).

1 Key knowledge and worked examples

Sampling words. A census measures the whole population: complete, no sampling error, but slow, costly, possibly destructive. A sample survey is cheap and fast but carries sampling error. The sampling frame lists sampling units. A statistic is a function of the sample values ONLY (contains no unknown parameter); its distribution across repeated samples is the sampling distribution.

Worked example – a score distribution (WST02 June 2025 Q4). Three beads from a 2:5 black:white bag score 4 per black, 1 per white: with $p_b = \frac{2}{7}$, X takes 3, 6, 9, 12 with probabilities $\frac{125}{343}, \frac{150}{343}, \frac{60}{343}, \frac{8}{343}$ (binomial weights for 0, 1, 2, 3 blacks). The number Y of 9-point sets in n plays is $B(n, \frac{60}{343})$, and $1 - (\frac{283}{343})^n > 0.95$ needs $n = 16$.

2 Practice

2.1 State one advantage and one disadvantage of a sample survey compared with a census. [2]

2.2 For samples (X_1, X_2) from a population with mean μ, state which of $X_1 + 2X_2$, $\bar{X} - \mu$ and $3\bar{X}$ are statistics, with reasons. [3]

2.3 A fair coin is tossed twice; S = number of heads. Write out the sampling distribution of S. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WST02 June 2025 Q4.)* A bag contains a large number of beads of which the ratio of black to white beads is 2:5. A player takes 3 beads at random from the bag and scores 4 points for each black bead and 1 point for each white bead. The random variable X is the total number of points scored. (a) Find the sampling distribution of X. (b) In a game a player takes n sets of 3 beads (each set replaced before the next); Y is the number of sets scoring exactly 9 points. Find the least n such that $P(Y \geq 1) > 0.95$. [9]

3.2 *(Original.)* Two fair dice are rolled; M is the larger score shown. (a) Find the sampling distribution of M. (b) Find $E(M)$. [7]