6.1 Kinematics and elastic strings

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS Hooke's law • elastic energy

Kinematics and elastic strings

Objective

Solve $a = f(x)$ problems via $v\,dv/dx$ with constants closed by initial conditions, and apply Hooke's law and elastic energy to strings and springs in equilibrium and in work-energy problems (objectives 1.1, 2.1, 2.2).

1 Key knowledge and worked examples

Two toolkits. $a = f(x)$: replace $\frac{dv}{dt}$ by $v\frac{dv}{dx}$, integrate, close the constant from a stated instant. Elastic string of natural length L, modulus λ, extension e: tension $\frac{\lambda e}{L}$, stored energy $\frac{\lambda e^2}{2L}$; each half of a cut string has half the natural length and half the extension.

Worked example – both (WME03 June 2025 Q3, Q4). (i) $a = 4x - 2$, at $t = 1$: $x = 0$, $v = 1$: $v\frac{dv}{dx} = 4x - 2$ gives $v^2 = (2x-1)^2$ so $v = 1 - 2x$; then $\frac{dv}{dt} = -2v$ gives $v = e^{2-2t}$. (ii) A string ($\lambda = mg$, natural $3a$) spans AB $= 4a$; P at the midpoint drops $\frac{3a}{2}$: each half has natural length $\frac{3a}{2}$ and length $\frac{5a}{2}$ at D, so the elastic energy gain is $\frac{mg}{3a} \cdot \frac{3a^2}{4} = \frac{mga}{4}$ per half – $\frac{1}{2}mga$ in total – and the work-energy equation with resistance $\frac{1}{5}mg$ gives $V = \sqrt{29ag/10}$.

2 Practice

2.1 A particle has $a = 6x$ and passes $x = 1$ with $v = 2$. Find v^2 in terms of x. [3]

2.2 An elastic string has natural length 1.2 m and modulus 24 N. Find the tension when the extension is 0.3 m, and the energy stored. [3]

2.3 A spring of natural length L and modulus λ hangs vertically with a mass m in equilibrium. Show that the extension is mgL/λ. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WME03 June 2025 Q3.)* A particle P moves along the x-axis. At time t seconds ($t \geq 1$) the displacement of P from O is x metres (positive x direction), where $x < \frac{1}{2}$; the velocity is v m s^{-1} and the acceleration a m s^{-2} (both positive x direction). Given $a = 4x - 2$ and that at $t = 1$, $x = 0$ and $v = 1$: (a) find v in terms of x; (b) find v in terms of t. [9]

3.2 *(Edexcel IAL WME03 June 2025 Q4.)* A light elastic string of natural length $3a$ has modulus of elasticity mg; its ends are attached to fixed points A and B with AB horizontal and $AB = 4a$. A particle P of mass m is attached to the midpoint of the string and held at rest at C, the midpoint of AB. P is projected vertically downwards from C with speed $\sqrt{3ag/2}$ and reaches D, where $CD = \frac{3}{2}a$, with speed V. (a) Show that the elastic energy stored in the string increases by $\frac{1}{2}mga$ as P moves from C to D. (b) Given the air resistance is a constant force of magnitude $\frac{1}{5}mg$, use the work-energy principle to find V in terms of a and g. [8]