# 5.4 Collisions and restitution

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

**KEY WORDS** coefficient of restitution • momentum

## Collisions and restitution

## Objective

Solve direct collisions with conservation of momentum and Newton's law of restitution, find the kinetic energy lost, and compute impulses between the colliding bodies (objectives 4.1–4.2).

## 1 Key knowledge and worked examples

**Two equations, two speeds.** Momentum: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (signed). Restitution: $e = \frac{v_B - v_A}{u_A - u_B}$ with $0 \le e \le 1$. The KE loss always carries the factor $(1-e^2)$; the impulse on a body is its own momentum change.

**Worked example – the full algebra (WME02 June 2025 Q5a–c).** P ($2m$, speed $u$) strikes Q ($m$, at rest): $2u = 2v_P + v_Q$ and $eu = v_Q - v_P$ give $v_Q = \frac{2(1+e)u}{3}$ and $v_P = \frac{(2-e)u}{3}$. KE after $= \frac{mu^2}{3}(2+e^2)$, so the loss is $\frac{mu^2}{3}(1-e^2)$ – the constant $k = \frac{1}{3}$. If the impulse on Q is $\frac{8mu}{9}$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9}$ gives $e = \frac{1}{3}$.

## 2 Practice

**2.1** A 3 kg particle at 4 $\mathrm{m\,s^{-1}}$ catches a 1 kg particle at 1 $\mathrm{m\,s^{-1}}$, both moving right; they coalesce. Find the common speed and the energy lost. [4]

**2.2** Two equal masses collide with $e = \frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]

**2.3** For a direct collision, show that the total kinetic energy is conserved when $e = 1$ is impossible unless the relative speed is zero – one line from the loss formula loss $= \frac{m_A m_B}{2(m_A+m_B)}(1-e^2)(u_A-u_B)^2$. [2]

## 3 Exam-style questions

**3.1** *(Edexcel IAL WME02 June 2025 Q5a–c, condensed.)* Two particles P (mass $2m$) and Q (mass $m$) lie at rest on a smooth horizontal plane. P is projected with speed $u$ directly towards Q, and they collide directly with coefficient of restitution $e$ ($e > 0$). (a) Show that the speed of Q immediately after the collision is $\frac{2(1+e)u}{3}$. (b) Show that the total kinetic energy lost in the collision is $k(1-e^2)mu^2$, where $k$ is a constant to be determined. (c) Given that the magnitude of the impulse exerted on Q by P is $\frac{8}{9}mu$, find $e$. [12]

**3.2** *(Original.)* A 2 kg particle moving right at 5 $\mathrm{m\,s^{-1}}$ collides directly with a 3 kg particle moving left at 1 $\mathrm{m\,s^{-1}}$; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]