3.1 Hyperbolic functions and equations

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS hyperbolic functions • exponential form

Hyperbolic functions and equations

Objective

Use the exponential definitions of cosh, sinh and tanh to derive identities, convert mixed equations into quadratics in e^x, and give exact logarithmic solutions (FP3 1.1).

1 Key knowledge and worked examples

Definitions. $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\tanh x = \dfrac{\sinh x}{\cosh x}$. Core identities: $\cosh^2 x - \sinh^2 x \equiv 1$, $\cosh 2x \equiv \cosh^2 x + \sinh^2 x \equiv 2\cosh^2 x - 1$, $\sinh 2x \equiv 2\sinh x \cosh x$. Every mixed equation $a\cosh x + b\sinh x = c$ becomes a quadratic in e^x; reject roots that make $e^x \leq 0$ and check survivors.

Worked example – identity then equation (WFM03 June 2025 Q1). From the definitions, $\cosh 6x + \cosh 4x = \frac{1}{2}(e^{6x} + e^{-6x}) + \frac{1}{2}(e^{4x} + e^{-4x}) = \frac{1}{2}(e^{5x} + e^{-5x})(e^x + e^{-x}) = 2\cosh 5x \cosh x$. So $\cosh 6x + \cosh 4x = 8\cosh x$ collapses to $\cosh x(2\cosh 5x - 8) = 0$, giving $\cosh 5x = 4$ and $x = \pm\frac{1}{5}\ln(4 + \sqrt{15})$.

2 Practice

2.1 Express $6\cosh x - 2\sinh x$ in the form $Ae^x + Be^{-x}$, and hence find its minimum value. [4]

2.2 Solve $\cosh 2x = 2$, giving your answers as natural logarithms in simplest form. [3]

2.3 Solve $5\cosh x + 3\sinh x = 4$. [4]

2.4 Solve $\tanh x = \frac{3}{5}$, giving your answer as a natural logarithm. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WFM03 June 2025 Q1.)* (a) Use the definition of $\cosh x$ in terms of exponentials to show that $\cosh 6x + \cosh 4x = 2\cosh 5x \cosh x$. (b) Hence determine the exact values of x for which $\cosh 6x + \cosh 4x = 8\cosh x$, giving your answers in terms of natural logarithms in simplest form. [6]

3.2 *(Original.)* (a) Use the definitions of sinh and cosh to show that $\sinh 2x = 2\sinh x \cosh x$.
(b) Hence solve $2\sinh x \cosh x = 3\sinh x$, giving exact answers in terms of natural logarithms. [5]