9.6 Mixed S3 practice

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS Central Limit Theorem

Mixed S3 practice

Objective

Combine the unit's tools on fuller problems: two-sample tests with variance recovery, estimator construction, chi-squared with parameter estimation, and pallet-type linear combinations (objectives 1.1, 3.1–3.8, 4.1–4.2).

1 Key knowledge and worked examples

Recurring chains. $\sum y$ and $\sum y^2 \to$ unbiased $\bar{y}$, $s_y^2 \to$ two-sample z. Litter counts $\to \hat{p} \to$ expected frequencies $\to$ X^2 with df lost to $\hat{p}$. A pallet problem rearranges the inequality until one Normal variable remains.

Worked example (WST03 June 2025 Q8c). With $S \sim N(18, 0.03^2)$, $C \sim N(25, 0.03^2)$, $P \sim N(15, 0.2^2)$: $T = \sum_1^{28} S_i + \sum_1^5 C_i + P_1$ and the event $T < 30P_1 + 190$ becomes $R = \sum S_i + \sum C_i - 29P_1 < 190$ where $R \sim N(194, 33.6697)$ – the pallet's coefficient 29 squares to 841 times its variance. Answer $\Phi\left(\frac{190-194}{5.804}\right) = 0.245$.

2 Practice

2.1 A sample of 50 has $\sum x = 350$ and $\sum x^2 = 2590$: find the unbiased estimates of μ and σ^2. [3]

2.2 Independent $X \sim N(40, 2)$ and $Y \sim N(37, 3)$: find $P(X > Y)$. [3]

2.3 In a goodness-of-fit test with 9 cells and two parameters estimated, four tail cells are combined into one: state the final degrees of freedom. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WST03 June 2025 Q8c.)* The weights of bags of sand are $S \sim N(18, 0.03^2)$ kg, bags of cement $C \sim N(25, 0.03^2)$ kg, and pallets $P \sim N(15, 0.2^2)$ kg, all independent. A load consists of 28 bags of sand, 5 bags of cement and a single pallet P_1. Find the probability that the total weight of the load is less than $30P_1 + 190$ kg, showing the distribution of every combination used. [6]

3.2 *(Original.)* A drinks machine's dispensing time is Normally distributed. A sample of 40 dispersions gives $\sum x = 92.0$ seconds and $\sum x^2 = 216.32$. (a) Find unbiased estimates of the mean and variance of the dispensing time. (b) Construct a 95% confidence interval for the mean time, justifying any substitution you make. (c) The manufacturer claims a mean of 2.2 seconds: test the claim at the 5% level (two-tailed) and conclude in context. [11]

Solutions

2.1 $\bar{x} = 7$; $s^2 = \frac{2590-50(49)}{49} = \frac{140}{49} = 2.857$.

2.2 $X - Y \sim N(3, 5)$: $z = \frac{3}{\sqrt{5}} = 1.342$, $P = 0.9103$ (awrt 0.910).

2.3 9 cells $\to$ 6 after combining; df $= 6 - 1 - 2 = 3$.

3.1 Let $R = \sum_{i=1}^{28} S_i + \sum_{i=1}^{5} C_i - 29P_1$. Then $\mathrm{E}(R) = 28(18)+5(25)-29(15) = 504+125-435 = 194$ and $\mathrm{Var}(R) = 28(0.0009) + 5(0.0009) + 29^2(0.04) = 0.0252 + 0.0045 + 33.64 = 33.6697$, so $R \sim N(194, 33.6697)$. The event is $R < 190$: $z = \frac{190-194}{\sqrt{33.6697}} = \frac{-4}{5.804} = -0.689$, so $P = 1 - 0.7549 = 0.245$ (awrt 0.245).

3.2 (a) $\bar{x} = \frac{92.0}{40} = 2.3$ s; $s^2 = \frac{216.32-40(2.3)^2}{39} = \frac{216.32-211.6}{39} = \frac{4.72}{39} = 0.121$. (b) With $n = 40$ large, substitute s^2 for σ^2: SE $= \sqrt{0.121/40} = 0.055$, so the interval is $2.3 \pm 1.96(0.055) = 2.3 \pm 0.108 = (2.192, 2.408)$ s. (c) $\mathrm{H}_0 : \mu = 2.2$ vs $\mathrm{H}_1 : \mu \neq 2.2$: $z = \frac{2.3-2.2}{0.055} = 1.82$ against two-tailed critical ± 1.96. Since $1.82 < 1.96$, do not reject H_0 – insufficient evidence, at the 5% level, against the manufacturer's claimed mean of 2.2 seconds (the true mean may still differ; 2.2 is simply consistent with the sample).

Local review guidance: the inequality rearranged before any distribution is written; the pallet coefficient squared in the variance; large-sample substitution justified by n; two-tailed comparison with ± 1.96 and a contextual conclusion. This is authored classroom guidance.