9.3 Confidence intervals and tests for means

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS confidence interval • critical value

Confidence intervals and tests for means

Objective

Build and interpret z-based confidence intervals, run one- and two-sample tests for a mean (including a non-zero null gap), and state the variance/CLT assumptions (objectives 3.3–3.8).

1 Key knowledge and worked examples

Formulas. CI: $\bar{x} \pm z\,\sigma/\sqrt{n}$. One-sample test: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$. Two samples with a gap: $z = \dfrac{(\bar{x}-\bar{y})-(\mu_x-\mu_y)}{\sqrt{\sigma_x^2/n_x+\sigma_y^2/n_y}}$. Large samples: substitute s^2 for σ^2 (CLT for non-Normal populations).

Worked example (WST03 June 2025 Q4). Grey squirrels ($n = 80$, $\bar{x} = 500$, $s_x^2 = 1156$); red squirrels ($n = 80$, $\sum y = 23200$, $\sum y^2 = 6741351$): $\bar{y} = 290$, $s_y^2 = \frac{6741351-80(290)^2}{79} = 169$. Testing $\mathrm{H}_0\colon \mu_x - \mu_y = 200$ vs $\mathrm{H}_1\colon\ > 200$: $z = \frac{10}{\sqrt{265/16}} = 2.457 > 1.6449$ – reject H_0. Assumption: the sample variances equal the population variances.

2 Practice

2.1 A sample of 36 from a Normal population with $\sigma = 9$ gives $\bar{x} = 50$. Construct a 95% confidence interval for μ. [3]

2.2 For the same data, test $H_0: \mu = 52$ against $\mu < 52$ at the 5% level. [3]

2.3 State the two large-sample licences that let you use s in place of σ. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WST03 June 2025 Q4.)* A biologist claims that the mean weight of grey squirrels is 200 g more than the mean weight of red squirrels. A research student believes the mean weight of grey squirrels is greater than 200 g more. Random samples of 80 grey and 80 red squirrels are taken; the weights of the grey squirrels are denoted by x grams with $\bar{x} = 500$ and $s_x^2 = 1156$, and the weights of the red squirrels by y grams with $\sum y = 23200$ and $\sum y^2 = 6741351$. (a) Find unbiased estimates of the mean and variance of the red squirrels' weights. (b) Using a suitable hypothesis test at the 5% level of significance, test the research student's belief, stating the critical value. (c) State the assumption you made about the variances. [11]

3.2 *(Original.)* A machine fills bottles; the fill volumes are Normally distributed with standard deviation 4 ml. A sample of 49 bottles gives $\bar{x} = 503$ ml. (a) Construct a 99% confidence interval for the mean fill. (b) The label claims 500 ml: test at the 1% level whether the mean fill exceeds 500 ml, and state the conclusion in context. [7]

Solutions

2.1 $50 \pm 1.96 \times \frac{9}{\sqrt{36}} = 50 \pm 1.96 \times 1.5 = (47.06, 52.94)$.

2.2 $z = \frac{50-52}{1.5} = -1.33$ vs critical -1.6449: $-1.33 > -1.6449$, do not reject – insufficient evidence that $\mu < 52$.

2.3 The sample size is large enough that (i) $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ is approximately $N(0,1)$, and (ii) by the Central Limit Theorem the population itself need not be Normal.

3.1 (a) $\bar{y} = \frac{23200}{80} = 290$; $s_y^2 = \frac{6741351-80(290)^2}{79} = \frac{13351}{79} = 169$. (b) $\mathrm{H}_0\colon \mu_x - \mu_y = 200$, $\mathrm{H}_1\colon \mu_x - \mu_y > 200$. $z = \dfrac{(500-290)-200}{\sqrt{1156/80 + 169/80}} = \dfrac{10}{\sqrt{16.5625}} = 2.457$. One-tailed 5% critical value 1.6449. Since $2.457 > 1.6449$, reject H_0 – there is sufficient evidence, at the 5% level, that grey squirrels' mean weight is more than 200 g greater than red squirrels'. (c) The sample variances equal the corresponding population variances ($\sigma_x^2 = s_x^2$ and $\sigma_y^2 = s_y^2$).

3.2 (a) $503 \pm 2.5758 \times \frac{4}{\sqrt{49}} = 503 \pm 1.47 = (501.53, 504.47)$ ml. (b) $\mathrm{H}_0\colon \mu = 500$ vs $\mathrm{H}_1\colon \mu > 500$: $z = \frac{503-500}{4/7} = 5.25$ vs critical 2.3263. Reject H_0 – strong evidence the mean fill exceeds 500 ml (the machine overfills on average).

Local review guidance: the null gap subtracted in the numerator; combined standard error with both variances over their own n; critical value with the correct tail and the conclusion in context. This is authored classroom guidance.