

## 7.6 The Normal distribution

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 22 marks

**KEY WORDS** Normal distribution

### The Normal distribution

#### Objective

Standardise Normal variables in both directions, use symmetry, handle binomial-style "exactly  $r$  of  $n$ " follow-ups, and finish two-condition problems with simultaneous equations (Normal objective).

#### 1 Key knowledge and worked examples

**Standardising.**  $X \sim N(\mu, \sigma^2)$ :  $Z = \frac{X-\mu}{\sigma}$  is standard Normal. Forward: probability from the table. Backward:  $z$  from the table, then  $x = \mu + z\sigma$ . Symmetry:  $P(X < \mu - a) = P(X > \mu + a)$ . Exactly  $r$  of  $n$  independent items:  $\binom{n}{r}p^r(1-p)^{n-r}$ . Two conditions on one unknown: equate the two  $z$ -expressions.

**Worked example – bags of flour (WST01 June 2025 Q6).** Small bags  $X \sim N(502, 3^2)$ :  $P(X > 508) = P(Z > 2) = 0.0228$ ;  $P(496 < X < 508) = 1 - 2(0.0228) = 0.9544$ ; of 4 bags exactly 2 over 508 g:  $6(0.0228)^2(0.9772)^2 = 0.00298$ . Large bags  $Y \sim N(1024, \sigma^2)$  with 85th percentile 1038.51:  $\sigma = \frac{14.51}{1.0364} = 14$ . And  $P(X > k) = P(Y < 2k)$  gives  $\frac{k-502}{3} = \frac{1024-2k}{14}$ , so  $k = 505$  and  $p = P(Z > 1) = 0.159$ .

#### 2 Practice

**2.1**  $Z \sim N(0, 1)$ . Write down  $P(Z > 1.5)$  and hence  $P(Z < -1.5)$ . [2]

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**2.2**  $X \sim N(40, 6^2)$ . Find  $P(X > 46)$ . [3]

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**2.3**  $X \sim N(50, \sigma^2)$  and  $P(X > 56) = 0.1587$ . Find  $\sigma$ . [3]

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### 3 Exam-style questions

**3.1** (*Edexcel IAL WST01 June 2025 Q6, condensed.*) The weight  $X$  grams in a small bag of flour is  $N(502, 3^2)$ . (a) Find  $P(X > 508)$  and hence  $P(496 < X < 508)$ . (b) A random sample of 4 small bags is taken; find the probability that exactly 2 of them contain more than 508 g. (c) The weight  $Y$  grams in a large bag is  $N(1024, \sigma^2)$  and the 85th percentile of  $Y$  is 1038.51; show that  $\sigma = 14$  to the nearest gram. [9]

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**3.2** (*Edexcel IAL WST01 June 2025 Q6d.*) Given that  $P(X > k) = P(Y < 2k)$ , where  $X \sim N(502, 3^2)$  and  $Y \sim N(1024, 14^2)$ , find  $k$  and the common probability. [5]

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## Solutions

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**2.1**  $P(Z > 1.5) = 0.0668$ ; by symmetry  $P(Z < -1.5) = 0.0668$ .

**2.2**  $z = \frac{46-40}{6} = 1$ :  $P(X > 46) = P(Z > 1) = 0.1587$ .

**2.3**  $P\left(Z > \frac{56-50}{\sigma}\right) = 0.1587 \implies z = 1 \implies \sigma = 6$ .

**3.1** (a)  $P(X > 508) = P\left(Z > \frac{508-502}{3}\right) = P(Z > 2) = 0.0228$ . Since 496 and 508 sit symmetrically two SDs either side of 502,  $P(496 < X < 508) = 1 - 2(0.0228) = 0.9544$ . (b) With  $p = 0.0228$ :  $\binom{4}{2}p^2(1-p)^2 = 6(0.0228)^2(0.9772)^2 = 0.00298$ . (c) The 85th percentile has  $z = 1.0364$ :  $\frac{1038.51-1024}{\sigma} = 1.0364$  gives  $\sigma = \frac{14.51}{1.0364} = 14.0$ , i.e.  $\sigma = 14$  ✓.

**3.2** Standardise both:  $P(X > k) = P\left(Z > \frac{k-502}{3}\right)$  and  $P(Y < 2k) = P\left(Z < \frac{1024-2k}{14}\right)$ . For these two probabilities to be equal, the two  $z$ -values must be EQUAL – a right tail and a left tail carry the same probability exactly when they are the same distance from 0 in the same direction:  $\frac{k-502}{3} = \frac{1024-2k}{14}$ . Clearing:  $14k - 7028 = 3072 - 6k$ , so  $20k = 10100$  and  $k = 505$ . Then  $p = P\left(Z > \frac{505-502}{3}\right) = P(Z > 1) = 0.159$ .

Local review guidance: standardisations with the correct sign pairings (equal probabilities  $\implies$  equal  $z$  for a  $>$  and a  $<$  on opposite sides); exact table values with 4 dp tolerance; binomial term with the correct coefficient. This is authored classroom guidance.