6.4 Horizontal circular motion

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS angular speed • centripetal acceleration

Horizontal circular motion

Objective

Use angular speed and radial acceleration $r\omega^2 = v^2/r$ with vertical equilibrium for table circles, conical pendulums and similar contexts (objectives 4.1–4.3).

1 Key knowledge and worked examples

Two resolutions. Towards the centre: the net force provides $mr\omega^2$ (or $\frac{mv^2}{r}$). Vertically: no acceleration, so forces balance. The radius is the horizontal distance to the axis, not the string length.

Worked example (WME03 June 2025 Q2). P (mass m) on a smooth table, tethered to O above by a string of length a at 30° to the table, circling with $\omega = \sqrt{\frac{g}{2a}}$. Radius $= a\cos 30°$. Centrewards: $T\cos 30° = m(a\cos 30°)\omega^2 = ma\cos 30° \cdot \frac{g}{2a} \implies T = \frac{mg}{2}$. Vertically: $R = mg - T\sin 30° = mg - \frac{mg}{4} = \frac{3mg}{4}$.

2 Practice

2.1 A particle moves in a circle of radius 0.5 m at 2 revolutions per second. Find ω and the speed. [2]

2.2 A conical pendulum: string length l, angle 45° to the vertical, mass m. Find the angular speed in terms of g and l. [3]

2.3 For that pendulum, find the tension in terms of mg. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WME03 June 2025 Q2.)* One end of a light inextensible string of length a is attached to a fixed point O above a smooth horizontal table; the other end is attached to a particle P of mass m on the table. P moves on the table in a horizontal circle with constant angular speed $\sqrt{\frac{g}{2a}}$, the string taut and making a constant angle of $30°$ with the table. Find, in terms of m and g, the magnitude of the force exerted on P by the table. [7]

3.2 *(Original.)* A car of mass m travels at speed v round a circular bend of radius r banked at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$ and there is no sideways frictional force on the car. (a) Show that $v^2 = \frac{3gr}{4}$. (b) Find the magnitude of the normal reaction in terms of m and g. [6]

Solutions

2.1 $\omega = 4\pi \text{ rad s}^{-1}$; $v = 0.5 \times 4\pi = 2\pi \text{ m s}^{-1}$.

2.2 Centrewards: $T \sin 45° = m(l \sin 45°)\omega^2 \implies T = ml\omega^2$. Vertically: $T \cos 45° = mg$ $\implies T = mg\sqrt{2}$. So $\omega^2 = \frac{g\sqrt{2}}{l}$, $\omega = \sqrt{\frac{g\sqrt{2}}{l}} = 2^{1/4}\sqrt{\frac{g}{l}}$.

2.3 $T = mg\sqrt{2}$.

3.1 Radius of the circle: $a \cos 30°$. Newton's law towards the centre: $T \cos 30° = m(a \cos 30°)\omega^2 = ma \cos 30° \cdot \frac{g}{2a}$, so $T = \frac{mg}{2}$. Vertically (no acceleration): $R + T \sin 30° = mg$, so $R = mg - \frac{mg}{2} \cdot \frac{1}{2} = mg - \frac{mg}{4} = \frac{3mg}{4}$.

3.2 (a) No friction, so the only forces are the weight mg and the normal reaction R perpendicular to the banked surface. Resolving horizontally (centrewards) and vertically: $R \sin \theta = \frac{mv^2}{r}$ and $R \cos \theta = mg$. Dividing: $\tan \theta = \frac{v^2}{gr}$, so with $\tan \theta = \frac{3}{4}$, $v^2 = \frac{3gr}{4}$ ✓. (b) $R = \frac{mg}{\cos \theta}$ with $\cos \theta = \frac{4}{5}$ (3-4-5 triangle): $R = \frac{5}{4}mg$.

Local review guidance: the centreward resolution with the RADIUS (not the string length); vertical equilibrium including the normal reaction; answers in terms of m and g exactly. This is authored classroom guidance.