6.3 Simple harmonic motion

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS simple harmonic motion • period

Simple harmonic motion

Objective

Prove SHM by deriving $\ddot{x} = -\omega^2 x$ (equilibrium tensions first), quote period and amplitude results, and compute exact time fractions (objectives 3.2, 3.3).

1 Key knowledge and worked examples

SHM routine. Find the equilibrium point; at displacement x write the net force with Hooke's law for every string; collect terms in x: $\ddot{x} = -\omega^2 x$ proves SHM. Period $\frac{2\pi}{\omega}$; amplitude a; $v^2 = \omega^2(a^2 - x^2)$; $v_{max} = a\omega$; $x = a\cos\omega t$ (released from an end). Time fractions come from $|\sin\omega t|$ or $|\cos\omega t|$ inequalities.

Worked example (WME03 June 2025 Q7). Strings from P to A ($\lambda = mg$, natural $3L$) and B ($\lambda = 2mg$, natural $2L$), $AB = 7L$: equilibrium at E ($AE = 4.5L$, both tensions $\frac{mg}{2}$). At displacement x from E: $m\ddot{x} = \frac{2mg}{2L}(\frac{L}{2} - x) - \frac{mg}{3L}(\frac{3L}{2} + x) = -\frac{4mg}{3L}x$: SHM with $\omega^2 = \frac{4g}{3L}$, period $\pi\sqrt{\frac{3L}{g}}$; amplitude $\frac{L}{2}$ (from AC $= 5L$), so $v_{max} = \sqrt{\frac{gL}{3}}$; the time with $|v| \leq \sqrt{\frac{gL}{12}} = \frac{v_{max}}{2}$ is one third of a period, $\pi\sqrt{\frac{L}{3g}}$.

2 Practice

2.1 A particle on a smooth table is attached to a spring (λ, natural L) fixed at A, and rests at equilibrium at E. It is displaced a small distance and released: write down ω^2 and the period. [3]

2.2 For SHM with amplitude 0.4 m and $\omega = 3\ \text{rad}\,\text{s}^{-1}$, find the maximum speed and the speed when $x = 0.2$ m. [3]

2.3 For that oscillation, find the time per oscillation for which $|x| \leq 0.2$ m. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME03 June 2025 Q7.)* A particle P of mass m lies at rest on a smooth horizontal table. One end of a light elastic string of natural length $3L$ and modulus of elasticity mg is attached to P and the other to a fixed point A on the table; a second string of natural length $2L$ and modulus $2mg$ runs from P to a fixed point B, with A, P, B collinear and $AB = 7L$. P rests in equilibrium at E with $AE = 4.5L$. P is held at C on AB with $AC = 5L$ and released from rest. (a) Show that P moves with simple harmonic motion about E and find the period. (b) Find the maximum speed of P. (c) Find the exact time, in any one oscillation, for which the speed of P is less than or equal to $\sqrt{gL/12}$. [16]

Solutions

2.1 $\omega^2 = \frac{\lambda}{mL}$; period $2\pi\sqrt{\frac{mL}{\lambda}}$.

2.2 $v_{max} = 0.4 \times 3 = 1.2\ \mathrm{m\,s^{-1}}$; $v = 3\sqrt{0.4^2 - 0.2^2} = 3\sqrt{0.12} = 1.04\ \mathrm{m\,s^{-1}}$ (3 sf).

2.3 $x = a\cos\omega t$, so $|x| \le \frac{a}{2}$ means $|\cos\omega t| \le \frac{1}{2}$: the phase sits in $[\frac{\pi}{3}, \frac{2\pi}{3}]$ modulo π – one third of each period: $\frac{1}{3} \times \frac{2\pi}{3} = \frac{2\pi}{9}$ s (about 0.70 s).

3.1 (a) Equilibrium check at E: A-string extension $4.5L - 3L = 1.5L$, tension $\frac{mg(1.5L)}{3L} = \frac{mg}{2}$; B-string extension $2.5L - 2L = 0.5L$, tension $\frac{2mg(0.5L)}{2L} = \frac{mg}{2}$ ✓. At displacement x from E towards B: $m\ddot{x} = \frac{2mg}{2L}\left(\frac{L}{2} - x\right) - \frac{mg}{3L}\left(\frac{3L}{2} + x\right) = \frac{mg}{L}\left(\frac{L}{2} - x - \frac{L}{2} - \frac{x}{3}\right) = -\frac{4mg}{3L}x$. So P moves with SHM about E with $\omega^2 = \frac{4g}{3L}$, and the period is $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3L}{4g}} = \pi\sqrt{\frac{3L}{g}}$. (b) Released from C: amplitude $= 5L - 4.5L = \frac{L}{2}$; $v_{max} = \frac{L}{2}\sqrt{\frac{4g}{3L}} = \sqrt{\frac{gL}{3}}$. (c) The threshold $\sqrt{\frac{gL}{12}}$ is exactly half of v_{max}, so $|v| \le \frac{v_{max}}{2} \iff |\cos\omega t| \le \frac{1}{2} \iff$ the phase lies in $[\frac{\pi}{3}, \frac{2\pi}{3}]$ modulo π – one third of every period: $t = \frac{1}{3}\pi\sqrt{\frac{3L}{g}} = \pi\sqrt{\frac{L}{3g}}$.

Local review guidance: the $\ddot{x} = -\omega^2 x$ derivation with the equilibrium check visible; amplitude read from the release point; (dM1) time fractions via the sinusoid inequality, exact throughout. This is authored classroom guidance.