5.4 Collisions and restitution

Name: _____ Class: _____ Date: _____ **Total: 27 marks**

KEY WORDS coefficient of restitution • momentum

Collisions and restitution

Objective

Solve direct collisions with conservation of momentum and Newton's law of restitution, find the kinetic energy lost, and compute impulses between the colliding bodies (objectives 4.1–4.2).

1 Key knowledge and worked examples

Two equations, two speeds. Momentum: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (signed). Restitution: $e = \frac{v_B - v_A}{u_A - u_B}$ with $0 \le e \le 1$. The KE loss always carries the factor $(1-e^2)$; the impulse on a body is its own momentum change.

Worked example – the full algebra (WME02 June 2025 Q5a–c). P ($2m$, speed u) strikes Q (m, at rest): $2u = 2v_P + v_Q$ and $eu = v_Q - v_P$ give $v_Q = \frac{2(1+e)u}{3}$ and $v_P = \frac{(2-e)u}{3}$. KE after $= \frac{mu^2}{3}(2+e^2)$, so the loss is $\frac{mu^2}{3}(1-e^2)$ – the constant $k = \frac{1}{3}$. If the impulse on Q is $\frac{8mu}{9}$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9}$ gives $e = \frac{1}{3}$.

2 Practice

2.1 A 3 kg particle at 4 $\mathrm{m\,s^{-1}}$ catches a 1 kg particle at 1 $\mathrm{m\,s^{-1}}$, both moving right; they coalesce. Find the common speed and the energy lost. [4]

2.2 Two equal masses collide with $e = \frac{1}{2}$: A at 6 $\mathrm{m\,s^{-1}}$ hits B at rest. Find both speeds after. [3]

2.3 For a direct collision, show that the total kinetic energy is conserved when $e = 1$ is impossible unless the relative speed is zero – one line from the loss formula loss $= \frac{m_A m_B}{2(m_A+m_B)}(1-e^2)(u_A-u_B)^2$. [2]

3 Exam-style questions

3.1 *(Edexcel IAL WME02 June 2025 Q5a–c, condensed.)* Two particles P (mass $2m$) and Q (mass m) lie at rest on a smooth horizontal plane. P is projected with speed u directly towards Q, and they collide directly with coefficient of restitution e ($e > 0$). (a) Show that the speed of Q immediately after the collision is $\frac{2(1+e)u}{3}$. (b) Show that the total kinetic energy lost in the collision is $k(1-e^2)mu^2$, where k is a constant to be determined. (c) Given that the magnitude of the impulse exerted on Q by P is $\frac{8}{9}mu$, find e. [12]

3.2 *(Original.)* A 2 kg particle moving right at 5 $\mathrm{m\,s^{-1}}$ collides directly with a 3 kg particle moving left at 1 $\mathrm{m\,s^{-1}}$; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]

Solutions

2.1 $3(4)+1(1) = 4v$: $v = 3.25\ \mathrm{m\,s^{-1}}$. KE before $= 24+0.5 = 24.5$ J; after $= \frac{1}{2}(4)(3.25)^2 = 21.125$ J; lost $= 3.375$ J.

2.2 $6 = v_A + v_B$ and $\frac{1}{2}(6) = v_B - v_A$: $v_B = 4.5$, $v_A = 1.5\ \mathrm{m\,s^{-1}}$ (A still moves forward, slower).

2.3 The loss is zero exactly when $(1-e^2)(u_A - u_B)^2 = 0$: either $e = 1$ (perfectly elastic) or the particles were already moving together ($u_A = u_B$, no collision).

3.1 (a) Momentum: $2mu = 2mv_P + mv_Q$; restitution: $eu = v_Q - v_P$. Substituting $v_P = v_Q - eu$: $2u = 2v_Q - 2eu + v_Q = 3v_Q - 2eu$, so $v_Q = \frac{2u(1+e)}{3}$ ✓. (b) From (a), $v_P = \frac{2(1+e)u}{3} - eu = \frac{(2-e)u}{3}$. KE after $= \frac{1}{2}(2m)\left(\frac{(2-e)u}{3}\right)^2 + \frac{1}{2}m\left(\frac{2(1+e)u}{3}\right)^2 = \frac{mu^2}{9}\left[(2-e)^2 + 2(1+e)^2\right] = \frac{mu^2}{9}(6+3e^2) = \frac{mu^2}{3}(2+e^2)$. Loss $= \frac{1}{2}(2m)u^2 - \frac{mu^2}{3}(2+e^2) = mu^2 - \frac{2mu^2}{3} - \frac{e^2mu^2}{3} = \frac{1}{3}(1-e^2)mu^2$, so $k = \frac{1}{3}$ ✓. (c) Impulse on Q $= m\Delta v$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9} \Rightarrow \frac{2(1+e)}{3} = \frac{8}{9} \Rightarrow 1 + e = \frac{4}{3}$, so $e = \frac{1}{3}$.

3.2 Right positive: momentum $2(5) + 3(-1) = 7 = 2v_A + 3v_B$; restitution $e = \frac{v_B - v_A}{5-(-1)} = \frac{v_B - v_A}{6} = \frac{1}{4}$, so $v_B = v_A + 1.5$. Then $2v_A + 3v_A + 4.5 = 7$: $v_A = 0.5$, $v_B = 2\ \mathrm{m\,s^{-1}}$ (both right). KE before $= 25 + 1.5 = 26.5$ J; after $= 0.25 + 6 = 6.25$ J; lost $= 20.25$ J.

Local review guidance: both equations with consistent signs; both speeds fully simplified before any energy statement; loss factorised to show the $(1-e^2)$ factor. This is authored classroom guidance.