

5.4 Collisions and restitution

Name: _____ Class: _____ Date: _____

Total: 27 marks

KEY WORDS coefficient of restitution • momentum

Collisions and restitution

Objective

Solve direct collisions with conservation of momentum and Newton's law of restitution, find the kinetic energy lost, and compute impulses between the colliding bodies (objectives 4.1–4.2).

1 Key knowledge and worked examples

Two equations, two speeds. Momentum: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (signed). Restitution: $e = \frac{v_B - v_A}{u_A - u_B}$ with $0 \leq e \leq 1$. The KE loss always carries the factor $(1 - e^2)$; the impulse on a body is its own momentum change.

Worked example – the full algebra (WME02 June 2025 Q5a–c). P ($2m$, speed u) strikes Q (m , at rest): $2u = 2v_P + v_Q$ and $eu = v_Q - v_P$ give $v_Q = \frac{2(1+e)u}{3}$ and $v_P = \frac{(2-e)u}{3}$. KE after $= \frac{mu^2}{3}(2 + e^2)$, so the loss is $\frac{mu^2}{3}(1 - e^2)$ – the constant $k = \frac{1}{3}$. If the impulse on Q is $\frac{8mu}{9}$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9}$ gives $e = \frac{1}{3}$.

2 Practice

2.1 A 3 kg particle at 4 m s^{-1} catches a 1 kg particle at 1 m s^{-1} , both moving right; they coalesce. Find the common speed and the energy lost. [4]

[3]

[2]

[12]

3.2 (*Original.*) A 2 kg particle moving right at 5 m s^{-1} collides directly with a 3 kg particle moving left at 1 m s^{-1} ; the coefficient of restitution is $\frac{1}{4}$. Find both velocities after the collision, and the total energy lost. [6]

Solutions

2.1 $3(4) + 1(1) = 4v$: $v = 3.25 \text{ m s}^{-1}$. KE before $= 24 + 0.5 = 24.5 \text{ J}$; after $= \frac{1}{2}(4)(3.25)^2 = 21.125 \text{ J}$; lost $= 3.375 \text{ J}$.

2.2 $6 = v_A + v_B$ and $\frac{1}{2}(6) = v_B - v_A$: $v_B = 4.5$, $v_A = 1.5 \text{ m s}^{-1}$ (A still moves forward, slower).

2.3 The loss is zero exactly when $(1 - e^2)(u_A - u_B)^2 = 0$: either $e = 1$ (perfectly elastic) or the particles were already moving together ($u_A = u_B$, no collision).

3.1 (a) Momentum: $2mu = 2mv_P + mv_Q$; restitution: $eu = v_Q - v_P$. Substituting $v_P = v_Q - eu$: $2u = 2v_Q - 2eu + v_Q = 3v_Q - 2eu$, so $v_Q = \frac{2u(1+e)}{3}$ ✓. (b) From (a), $v_P = \frac{2(1+e)u}{3} - eu = \frac{(2-e)u}{3}$. KE after $= \frac{1}{2}(2m) \left(\frac{(2-e)u}{3} \right)^2 + \frac{1}{2}m \left(\frac{2(1+e)u}{3} \right)^2 = \frac{mu^2}{9} [(2-e)^2 + 2(1+e)^2] = \frac{mu^2}{9} (6 + 3e^2) = \frac{mu^2}{3} (2 + e^2)$. Loss $= \frac{1}{2}(2m)u^2 - \frac{mu^2}{3}(2 + e^2) = mu^2 - \frac{2mu^2}{3} - \frac{e^2mu^2}{3} = \frac{1}{3}(1 - e^2)mu^2$, so $k = \frac{1}{3}$ ✓. (c) Impulse on Q $= m\Delta v$: $m \cdot \frac{2(1+e)u}{3} = \frac{8mu}{9} \Rightarrow \frac{2(1+e)}{3} = \frac{8}{9} \Rightarrow 1 + e = \frac{4}{3}$, so $e = \frac{1}{3}$.

3.2 Right positive: momentum $2(5) + 3(-1) = 7 = 2v_A + 3v_B$; restitution $e = \frac{v_B - v_A}{5 - (-1)} = \frac{v_B - v_A}{6} = \frac{1}{4}$, so $v_B = v_A + 1.5$. Then $2v_A + 3v_A + 4.5 = 7$: $v_A = 0.5$, $v_B = 2 \text{ m s}^{-1}$ (both right). KE before $= 25 + 1.5 = 26.5 \text{ J}$; after $= 0.25 + 6 = 6.25 \text{ J}$; lost $= 20.25 \text{ J}$.

Local review guidance: both equations with consistent signs; both speeds fully simplified before any energy statement; loss factorised to show the $(1 - e^2)$ factor. This is authored classroom guidance.