5.2 Centres of mass

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS centre of mass • wire framework

Centres of mass

Objective

Find centres of mass of particle systems, uniform laminas, composite shapes (negative areas) and wire frameworks (length weighting with signed distances), and use suspension geometry (objectives 2.1–2.3).

1 Key knowledge and worked examples

Table method. Weight each piece by mass – for a uniform lamina by AREA, for a wire by LENGTH – and take moments about a convenient line: $\sum(\text{weight})(\bar{x}_{\text{piece}}) = (\text{total})(\bar{x})$. Holes and opposite-side pieces enter NEGATIVE. Booklet results: semicircular wire $\frac{2r}{\pi}$, semicircular lamina $\frac{4r}{3\pi}$ from the centre. Suspended from a point, the body hangs with G vertically below it; the asked angle usually splits a triangle angle at the suspension point.

Worked example – an S-shaped framework (WME02 June 2025 Q3). Arcs AB ($r = 2a$, length $2\pi a$, COM $\frac{4a}{\pi}$ one side) and CD ($r = a$, length πa, COM $\frac{2a}{\pi}$ the OTHER side) joined by straights $AC = DB = a$ on the line: moments about the line give $2\pi a \cdot \frac{4a}{\pi} - \pi a \cdot \frac{2a}{\pi} = 6a^2$; total length $(3\pi + 2)a$; so $\bar{x} = \frac{6a}{3\pi+2}$. Suspended from A with E on arc AB at $\angle AOE = 90°$: G lies on OE, $\tan\angle OAG = \frac{3}{3\pi+2}$, and $\theta = 45° - \angle OAG$ (since $OA = OE$), giving $\tan\theta = \frac{3\pi-1}{3\pi+5}$.

2 Practice

2.1 Particles of mass 2 kg, 3 kg and 5 kg sit at $x = 0$, 4 and 6 on the x-axis. Find their centre of mass. [2]

2.2 A uniform rectangular lamina 6 m by 4 m has a 2 m by 2 m square hole cut from one corner. Find the distance of the centre of mass from each outer edge adjacent to that corner. [4]

2.3 Three uniform rods form a uniform wire triangle with sides 5, 5 and 6. Find the distance of the centre of mass from the 6-side. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME02 June 2025 Q3, condensed.)* A uniform rigid wire framework consists of two semicircular arcs AB and CD, both of centre O, joined by two straight pieces AC and DB. The arc AB has radius $2a$, the arc CD has radius a, $AC = DB = a$, and ACDB is a straight line. The arcs lie on opposite sides of ACDB. The centre of mass of a semicircular arc of radius r is $\frac{2r}{\pi}$ from its centre. (a) Show that the centre of mass of the framework is $\frac{6a}{3\pi+2}$ from AB. (b) The point E lies on the arc AB with $\angle AOE = 90°$. The framework is freely suspended from A and hangs in equilibrium with AE at angle θ to the downward vertical. Show that $\tan\theta = \dfrac{3\pi - 1}{3\pi + 5}$. [10]

3.2 *(Original.)* A uniform semicircular lamina of radius 6 cm is freely suspended from one end P of its straight edge. Find the angle between the straight edge and the vertical, giving an exact tangent. [4]

Solutions

2.1 $\bar{x} = \frac{2(0)+3(4)+5(6)}{10} = \frac{42}{10} = 4.2$ from the 2 kg particle.

2.2 Rectangle (area 24) at (3, 2); hole (area -4) at (1, 1) from the adjacent corner: $\bar{x} = \frac{24(3)-4(1)}{20} = \frac{68}{20} = 3.4$ m and $\bar{y} = \frac{24(2)-4(1)}{20} = \frac{44}{20} = 2.2$ m from the two outer edges at that corner.

2.3 The two 5-sides each have midpoint 1.5 m above the 6-side (height of the 5-5-6 triangle = 4); the 6-side has COM distance 0. Weights by length: $\bar{y} = \frac{5(1.5)+5(1.5)+6(0)}{16} = \frac{15}{16} = 0.9375$ m from the 6-side.

3.1 (a) Table about the line AB: arc AB length $2\pi a$, COM $\frac{4a}{\pi}$ above; arc CD length πa, COM $\frac{2a}{\pi}$ BELOW; straights total $2a$ on the line: $2\pi a \cdot \frac{4a}{\pi} - \pi a \cdot \frac{2a}{\pi} + 2a \cdot 0 = 8a^2 - 2a^2 = 6a^2$; total length $= (3\pi+2)a$; so $\bar{x} = \frac{6a^2}{(3\pi+2)a} = \frac{6a}{3\pi+2}$ ✓ (the COM is on the AB side, distance d from the line). (b) By symmetry G lies on OE. In triangle AOE, $OA = OE = 2a$ and the angle at O is 90°, so $\angle OAE = 45°$; the COM is at distance d from A's vertical: in right triangle AOG, $\tan\angle OAG = \frac{d}{2a} = \frac{6}{2(3\pi+2)} = \frac{3}{3\pi+2}$. Hanging from A puts G on the vertical, so $\theta = \angle GAE = \angle OAE - \angle OAG = 45° - \angle OAG$, and $\tan\theta = \dfrac{1 - \frac{3}{3\pi+2}}{1 + \frac{3}{3\pi+2}} = \dfrac{3\pi - 1}{3\pi + 5}$ ✓.

3.2 The COM sits on the symmetry axis at $\frac{4r}{3\pi} = \frac{8}{\pi}$ cm from the centre O, i.e. $\frac{8}{\pi}$ from the straight edge, on the perpendicular through O. Suspended from P (one end of the edge, $OP = 6$), G hangs below P. In the right triangle with legs $OP = 6$ (along the edge) and the COM's perpendicular distance $\frac{8}{\pi}$ from the edge: the angle θ between the edge and the vertical satisfies $\tan\theta = \dfrac{8/\pi}{6} = \dfrac{4}{3\pi}$.

Local review guidance: pieces weighted by area/length with the correct booklet distance; opposite-side pieces signed; suspension angles via right-triangle trigonometry at the suspension point plus isosceles geometry. This is authored classroom guidance.