5.1 Kinematics with variable acceleration

Name: _____ Class: _____ Date: _____ **Total: 26 marks**

KEY WORDS integration • impulse

Kinematics with variable acceleration

Objective

Integrate acceleration as a function of time to velocity and displacement (constants from initial conditions), handle total distance via turning points, and differentiate position vectors before applying impulse-momentum (objectives 1.1–1.4).

1 Key knowledge and worked examples

Calculus kinematics. $v = \int a\,dt$, $s = \int v\,dt$: each integration constant is fixed by a stated instant ("at rest at O" kills both). Total DISTANCE with a reversal: find t from $v = 0$, compute each leg, add. Vectors: $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ component-wise; impulse $\mathbf{I} = m(\mathbf{v}' - \mathbf{v})$ at the stated instant.

Worked example – both halves (WME02 June 2025 Q1, Q2). (i) $a = k(3-t^2)$ from rest at O, $v(1.5) = 13.5$: $v = k(3t - \frac{t^3}{3})$ with zero constant, and $3.375k = 13.5$ gives $k = 4$; $s = 6t^2 - \frac{t^4}{3}$. $v = 0$ again at $t = 3$ with $s(3) = 27$, so the total distance is 54 m. (ii) $\mathbf{r} = 4\sqrt{t}\,\mathbf{i} - 3t\mathbf{j}$, mass 3 kg: $\mathbf{v} = 2t^{-1/2}\mathbf{i} - 3\mathbf{j} = \mathbf{i} - 3\mathbf{j}$ at $t = 4$; an impulse $(6\mathbf{i} - 3\mathbf{j})$ N s gives $\mathbf{v}' = 3\mathbf{i} - 4\mathbf{j}$, speed 5 m s^{-1}.

2 Practice

2.1 A particle moves with $a = 6t$ from rest at O. Find v and s when $t = 2$. [3]

2.2 With $v = 4t - t^2$ (m s^{-1}) from O, find the times when the particle is at rest and the total distance travelled between those times. [4]

2.3 $\mathbf{r} = (2t^2 + 1)\mathbf{i} + 3t\mathbf{j}$ metres: find the speed when $t = 3$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME02 June 2025 Q1.)* A particle P moves along the x-axis. At time $t = 0$, P is at rest at the origin O. At time t seconds the acceleration of P in the positive x direction is $k(3 - t^2)$ m s^{-2}, where k is a positive constant. Given that when $t = 1.5$ the speed of P is 13.5 m s^{-1}, (a) show that $k = 4$, (b) show that $s = \frac{1}{3}t^2(18 - t^2)$, (c) find the total distance travelled by P before it returns to O. [9]

3.2 *(Edexcel IAL WME02 June 2025 Q2.)* A particle P of mass 3 kg moves on a smooth horizontal plane. Relative to a fixed origin, $\mathbf{r} = 4t^{\frac{1}{2}}\mathbf{i} - 3t\mathbf{j}$ metres for $1 \leq t \leq 4$. At the instant $t = 4$, P receives an impulse $(6\mathbf{i} - 3\mathbf{j})$ N s. Find the speed of P immediately after the impulse. [7]

Solutions

2.1 $v = 3t^2 = 12\ \mathrm{m\,s^{-1}}$; $s = t^3 = 8$ m.

2.2 $v = 0$ at $t = 0$ and $t = 4$. $s = \int_0^4 (4t - t^2)dt = [2t^2 - \frac{t^3}{3}]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$ m; on $[0, 4]$ the velocity stays non-negative, so the distance equals the displacement, $\frac{32}{3}$ m (≈ 10.7 m).

2.3 $\mathbf{v} = 4t\mathbf{i} + 3\mathbf{j} = 12\mathbf{i} + 3\mathbf{j}$ at $t = 3$: speed $\sqrt{144 + 9} = \sqrt{153} = 12.4\ \mathrm{m\,s^{-1}}$.

3.1 (a) $v = \int k(3 - t^2)dt = k(3t - \frac{t^3}{3}) + C$; at rest at $t = 0$ gives $C = 0$. Then $k(3(1.5) - \frac{1.5^3}{3}) = k(4.5 - 1.125) = 3.375k = 13.5$, so $k = 4$ ✓. (b) $s = \int 4(3t - \frac{t^3}{3})dt = 6t^2 - \frac{t^4}{3} + D$ with $D = 0$ (at O when $t = 0$), i.e. $s = \frac{1}{3}t^2(18 - t^2)$ ✓. (c) $v = \frac{4t}{3}(9 - t^2) = 0$ at $t = 3$; $s(3) = 6(9) - \frac{81}{3} = 54 - 27 = 27$. The particle returns to O, so the total distance is $27 + 27 = 54$ m.

3.2 $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2t^{-\frac{1}{2}}\mathbf{i} - 3\mathbf{j}$; at $t = 4$: $2(4)^{-\frac{1}{2}} = 1$, so $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$. Impulse-momentum: $3(\mathbf{v}' - (\mathbf{i} - 3\mathbf{j})) = (6\mathbf{i} - 3\mathbf{j})$, so $\mathbf{v}' = (\mathbf{i} - 3\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) = 3\mathbf{i} - 4\mathbf{j}$. Speed $= \sqrt{9 + 16} = 5\ \mathrm{m\,s^{-1}}$.

Local review guidance: constants of integration closed by a stated instant before any substitution; turning point found from $v = 0$ for total distance; vector differentiation BEFORE the impulse. This is authored classroom guidance.