4.6 Statics of a particle and moments

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS equilibrium • moment of a force

Statics of a particle and moments

Objective

Resolve for equilibrium in two directions (strings, caps at μR), and take moments about well-chosen pivots for rods and tilting bodies (objectives 5.1–5.3, 6.1).

1 Key knowledge and worked examples

Equilibrium. Resolve horizontally and vertically (or along/perpendicular to a slope) and set both to zero; alternatively close the vector triangle. On rough ground at rest, friction takes whatever value is needed, up to μR – equality only at limiting equilibrium ("on the point of slipping"). **Moments**: force × perpendicular distance about a pivot, signed by sense; choose the pivot that eliminates unknowns; "tilting about X" means take moments about X with the other reaction gone.

Worked example – a rod on a ledge (WME01 June 2025 Q5). Rod AB of length $3a$ on a ledge, edge at C with $AC = 2a$, centre of mass d from A. Upward P at B, tilting about A: $3aP = Wd$. Downward $1.25P$ at B, tilting about C: $1.25Pa = W(2a - d)$. Substituting $P = \frac{Wd}{3a}$: $\frac{1.25d}{3} = 2a - d$, so $4.25d = 6a$ and $d = \frac{24a}{17}$.

2 Practice

2.1 A 50 N weight hangs from two light strings making 40° and 60° with the vertical. Find both tensions. [4]

2.2 A block rests in limiting equilibrium on a rough plane inclined at 25° to the horizontal. Find the coefficient of friction. [3]

2.3 A uniform beam of weight 300 N and length 6 m rests on supports at its ends. A person of weight 800 N stands 1 m from the left end. Find both reactions. [4]

2.4 A see-saw pivots at its centre. A child of mass 40 kg sits 2 m to the left of the pivot. How far from the pivot, on the other side, must a child of mass 30 kg sit for equilibrium? [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME01 June 2025 Q5.)* A non-uniform rod AB of length $3a$ rests in equilibrium on a horizontal ledge, with the edge of the ledge at the point C of the rod, where $AC = 2a$ and $CB = a$. The weight of the rod is W and its centre of mass is at distance d from A. A force of magnitude P is applied vertically upwards at B and the rod is on the point of tilting about A. This force is removed and a force of magnitude $1.25P$ is applied vertically downwards at B; the rod is now on the point of tilting about C. Find d in terms of a. [6]

3.2 *(Original.)* A non-uniform rod AB of length 4 m has weight W and centre of mass 1.5 m from A. It rests horizontally on supports at A and at C, where $AC = 3$ m. A load of weight $0.3W$ hangs from B. Find the reaction at each support. [5]

Solutions

2.1 Horizontal: $T_1 \sin 40° = T_2 \sin 60°$; vertical: $T_1 \cos 40° + T_2 \cos 60° = 50$. From the first, $T_2 = \frac{\sin 40°}{\sin 60°} T_1 = 0.742 T_1$; then $T_1(0.766 + 0.371) = 50$, so $T_1 = 44$ N and $T_2 = 33$ N (2 s.f.).

2.2 Along the slope: $mg \sin 25° = F$; perpendicular: $R = mg \cos 25°$. Limiting: $\mu = \frac{F}{R} = \tan 25° = 0.466$ (0.47).

2.3 Moments about the left support: $R_R(6) = 300(3) + 800(1)$, so $R_R = \frac{1700}{6} = 283$ N (280); vertically: $R_L = 1100 - 283 = 817$ N (820).

2.4 Moments about the pivot: $40g(2) = 30g\,d$, so $d = \frac{80}{30} = 2.7$ m ($\frac{8}{3}$ m).

3.1 Tilting about A (reaction at C zero): moments about A, $P(3a) = Wd$. Tilting about C (reaction at A zero): moments about C, $1.25P(a) = W(2a - d)$. Substituting $P = \frac{Wd}{3a}$ into the second: $1.25 \cdot \frac{Wd}{3a} \cdot a = W(2a - d)$, i.e. $\frac{1.25d}{3} = 2a - d$, so $1.25d = 6a - 3d$, $4.25d = 6a$, and $d = \frac{6a}{4.25} = \frac{24a}{17}$.

3.2 Moments about A: $R_C(3) = W(1.5) + 0.3W(4) = 1.5W + 1.2W = 2.7W$, so $R_C = 0.9W$. Total vertical: $R_A + R_C = W + 0.3W = 1.3W$, so $R_A = 0.4W$.

Local review guidance: two resolution equations (or moments about a killing pivot) with every force included; limiting friction only when the words license it; tilting answers as exact fractions with the pivot named per case. This is authored classroom guidance.