4.1 Vectors in mechanics

Name: _____ Class: _____ Date: _____ **Total: 25 marks**

KEY WORDS bearing • resultant

Vectors in mechanics

Objective

Work with displacements, velocities and forces as vectors in i, j form: magnitudes, resultants, position vectors at time t, relative position, and bearings (objectives 2.1–2.2).

1 Key knowledge and worked examples

Toolkit. $|a\mathbf{i} + b\mathbf{j}| = \sqrt{a^2 + b^2}$. Resultants add component-wise. Position at time t: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$ for constant velocity, built from any known instant. Bearings are clockwise from north: with $\mathbf{i}$ east and $\mathbf{j}$ north, the vector $x\mathbf{i} + y\mathbf{j}$ with $x < 0$, $y < 0$ points south-west, at $\tan^{-1}(|x/y|)$ west of due south.

Worked example – position vectors and a bearing (WME01 June 2025 Q2). A has $\mathbf{r}_A(2) = -5\mathbf{i} + 16\mathbf{j}$, $\mathbf{r}_A(5) = 10\mathbf{i} + 4\mathbf{j}$: velocity $= \frac{15\mathbf{i}-12\mathbf{j}}{3} = 5\mathbf{i} - 4\mathbf{j}$, and $\mathbf{r}_A(t) = (5t-15)\mathbf{i} + (24-4t)\mathbf{j}$. B sits at $-5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$; at $t = 5$, $\overrightarrow{AB} = -5\mathbf{i} - 12\mathbf{j}$, which is 22.6° west of south – bearing 203°.

2 Practice

2.1 Find the magnitude of the velocity $3\mathbf{i} - 4\mathbf{j}$ m s^{-1} and the bearing on which the particle moves. [3]

2.2 Forces $(2\mathbf{i} + \mathbf{j})$ N and $(4\mathbf{i} - 3\mathbf{j})$ N act on a particle. Find the magnitude and direction of the resultant force. [3]

2.3 A particle has position vector $3\mathbf{i} - 2\mathbf{j}$ m when $t = 1$ and $12\mathbf{i} + 7\mathbf{j}$ m when $t = 4$. Find its velocity and its position vector at time t seconds. [4]

2.4 Find the value of λ for which $\lambda\mathbf{i} + 2\mathbf{j}$ is perpendicular to $3\mathbf{i} - 4\mathbf{j}$. [3]

3 Exam-style questions

3.1 *(Edexcel IAL WME01 June 2025 Q2.)* At time t seconds, the position vector of particle A is $\mathbf{r}$ metres. When $t = 2$, $\mathbf{r} = (-5\mathbf{i} + 16\mathbf{j})$; when $t = 5$, $\mathbf{r} = (10\mathbf{i} + 4\mathbf{j})$. Particle B has position vector $\mathbf{s} = -5\mathbf{i} + 7\mathbf{j} + t(2\mathbf{i} - 3\mathbf{j})$ metres. (a) Find the velocity of A. (b) Find $\mathbf{r}$ in terms of t. (c) Find the bearing of B from A when $t = 5$, giving your answer to the nearest degree. [7]

3.2 *(Original.)* A ship leaves the point with position vector $(2\mathbf{i} - \mathbf{j})$ km and moves with constant velocity $(3\mathbf{i} + 4\mathbf{j})$ $\mathrm{km\,h^{-1}}$, where $\mathbf{i}$ and $\mathbf{j}$ point east and north. (a) Find the bearing of the ship from the origin at the moment it leaves. (b) Find t when the ship is due east of the origin, and its distance from the origin then. [5]

Solutions

2.1 Magnitude $\sqrt{9+16} = 5\ \mathrm{m\,s^{-1}}$. The direction $(3,-4)$ is south-east; $\tan^{-1}(3/4) = 36.9°$ east of south, so the bearing is $180 - 37 = 143°$.

2.2 Resultant $6\mathbf{i} - 2\mathbf{j}$: magnitude $\sqrt{36+4} = 2\sqrt{10} \approx 6.3$ N, direction $\tan^{-1}(2/6) = 18.4°$ south of east.

2.3 Velocity $= \dfrac{(12\mathbf{i}+7\mathbf{j}) - (3\mathbf{i}-2\mathbf{j})}{3} = 3\mathbf{i} + 3\mathbf{j}$; $\mathbf{r}(t) = (3\mathbf{i} - 2\mathbf{j}) + (t-1)(3\mathbf{i}+3\mathbf{j}) = 3t\mathbf{i} + (3t-5)\mathbf{j}$.

2.4 Perpendicular $\iff$ dot product zero: $3\lambda - 8 = 0$, so $\lambda = \frac{8}{3}$.

3.1 (a) $\dfrac{(10\mathbf{i}+4\mathbf{j}) - (-5\mathbf{i}+16\mathbf{j})}{3} = \dfrac{15\mathbf{i}-12\mathbf{j}}{3} = (5\mathbf{i}-4\mathbf{j})\ \mathrm{m\,s^{-1}}$. (b) $\mathbf{r} = (-5\mathbf{i}+16\mathbf{j}) + (t-2)(5\mathbf{i}-4\mathbf{j}) = (5t-15)\mathbf{i} + (24-4t)\mathbf{j}$. (c) At $t=5$: A is at $10\mathbf{i}+4\mathbf{j}$; B is at $-5\mathbf{i}+7\mathbf{j}+5(2\mathbf{i}-3\mathbf{j}) = 5\mathbf{i} - 8\mathbf{j}$; so $\overrightarrow{AB} = -5\mathbf{i} - 12\mathbf{j}$, magnitude 13, direction $\tan^{-1}(5/12) = 22.6°$ west of due south: bearing 203°.

3.2 (a) Position $(2,-1)$: south-east quadrant, $\tan^{-1}(2/1) = 63.4°$ east of south, bearing $180 - 63 = 117°$. (b) Position $(2+3t)\mathbf{i} + (-1+4t)\mathbf{j}$: due east when the $\mathbf{j}$-component is zero, $t = 0.25$ h; position $2.75\mathbf{i}$, so the ship is 2.75 km due east of the origin.

Local review guidance: difference quotient with the correct time interval; position vectors generalised from one instant; bearings measured clockwise from north in the correct quadrant. This is authored classroom guidance.