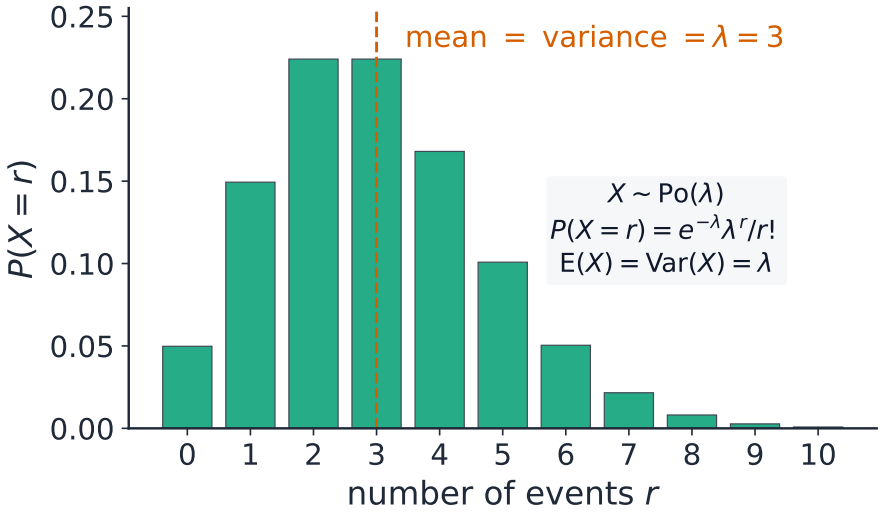
Exercise walk-through

8.4
Populations, samples and sampling distributions

Pearson Edexcel · International A-Level · Further Mathematics

Figure: The binomial and Poisson distributions



- The **Poisson distribution** $\mathrm{Po}(3)$: for a Poisson variable the mean and variance both equal λ.
- Read the axes and units first.

Continuous random variables (5)

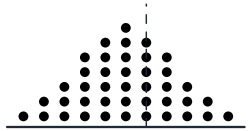
- Total area: $\frac{k^2}{21} + \frac{2}{15}\left(18 - 6k + \frac{k^2}{2}\right) = 1.$
- Multiply by 105: $5k^2 + 252 - 84k + 7k^2 = 105$, so $12k^2 - 84k + 147 = 0$, that is $4k^2 - 28k + 49 = 0$, or $(2k-7)^2 = 0$. So $k = 3.5$, and both parts equal $\frac{1}{3}$ there.
- $E(X) = \int_0^{3.5} \frac{2x^2}{21}\,dx + \int_{3.5}^{6} \frac{2(6x - x^2)}{15}\,dx = 1.361 + 1.806 = \frac{19}{6}.$
- With $E(X^2) = \frac{277}{24}$, $\mathrm{Var}(X) = \frac{277}{24} - \left(\frac{19}{6}\right)^2 = \frac{109}{72}.$

Populations, samples and statistics

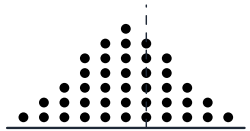
- A **census** measures every member of the population. It gives complete information, but it is slow and expensive, and impossible if testing destroys the item.
- To **sample** is to measure only part of the population. A **sample** is quicker and cheaper, but the results vary from sample to sample.
- A **sampling frame** is a list of the members of the population that the sample is chosen from.

Exercises 2.1·2.2

2.1 State one advantage and one disadvantage of a sample survey compared with a census. [2]

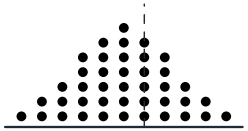


2.2 For samples (X_1, X_2) from a population with mean μ, state which of $X_1 + 2X_2$, $\bar{X} - \mu$ and $3\bar{X}$ are statistics, with reasons. [3]



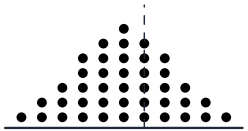
Exercises 2.1·2.2

2.1 State one advantage and one disadvantage of a sample survey compared with a census. [2]



2.1 Advantage: cheaper and faster than a census. Disadvantage: the results contain sampling error – they do not describe every member of the population.

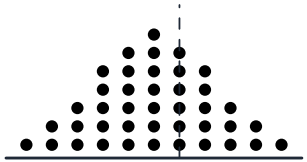
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2.2 $X_1 + 2X_2$ and $3\bar{X}$ involve sample values only: statistics. $\bar{X} - \mu$ contains the unknown population parameter μ: not a statistic.

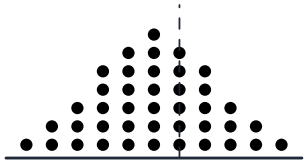
Exercise 2.3

2.3 A fair coin is tossed twice; S = number of heads. Write out the sampling distribution of S. [2]



Exercise 2.3

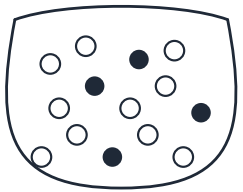
2.3 A fair coin is tossed twice; S = number of heads. Write out the sampling distribution of S. [2]



2.3 $P(S=0)=\frac{1}{4}$, $P(S=1)=\frac{1}{2}$, $P(S=2)=\frac{1}{4}$.

Exercise 3.1

3.1 *(Edexcel IAL WST02 June 2025 Q4.)* A bag contains a large number of beads of which the ratio of black to white beads is 2:5. A player takes 3 beads at random from the bag and scores 4 points for each black bead and 1 point for each white bead. The random variable X is the total number of points scored. (a) Find the sampling distribution of X. (b) In a game a player takes n sets of 3 beads (each set replaced before the next); Y is the number of sets scoring exactly 9 points. Find the least n such that $P(Y \geq 1) > 0.95$. [9]



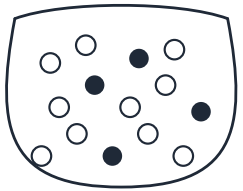
large bag (schematic)

- black: 4 points
- white: 1 point

black : white = 2 : 5

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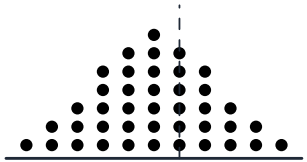
black : white = 2 : 5

3.1 (a) With $p_b = \frac{2}{7}$ (large bag: constant probability), the number of black beads is $B(3, \frac{2}{7})$ and X = 3 + 3 · (blacks): $P(X = 3) = (\frac{5}{7})^3 = \frac{125}{343}$; $P(X = 6) = \binom{3}{1}\frac{2}{7}(\frac{5}{7})^2 = \frac{150}{343}$; $P(X = 9) = \binom{3}{2}(\frac{2}{7})^2\frac{5}{7} = \frac{60}{343}$; $P(X = 12) = (\frac{2}{7})^3 = \frac{8}{343}$ (total $\frac{343}{343}$ ✓). (b) $Y \sim B(n, \frac{60}{343})$: $P(Y \geq 1) = 1 - (\frac{283}{343})^n > 0.95$, so $(\frac{283}{343})^n < 0.05$, $n \ln \frac{283}{343} < \ln 0.05$, $n > 15.6$: least $n = 16$.

Exercise 3.2

3.2 *(Original.)* Two fair dice are rolled; M is the larger score shown.

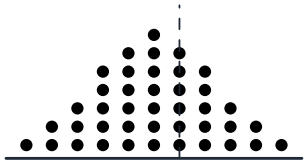
(a) Find the sampling distribution of M. (b) Find $E(M)$. [7]



Exercise 3.2

3.2 *(Original.)* Two fair dice are rolled; M is the larger score shown.

(a) Find the sampling distribution of M. (b) Find $E(M)$. [7]



3.2 (a) $P(M=m) = P(\max = m) = \left(\frac{m}{6}\right)^2 - \left(\frac{m-1}{6}\right)^2 = \frac{2m-1}{36}$ for $m = 1..6$: $\frac{1}{36}, \frac{3}{36}, \frac{5}{36}, \frac{7}{36}, \frac{9}{36}$, (sums to 1 ✓). (b) $E(M) = \sum m \cdot \frac{2m-1}{36}$: summing $m(2m-1) = 2m^2 - m$ over $m = 1..6$ gives $2(91) - 21 = 161$, so $E(M) = \frac{161}{36} = 4.47$.

Local review guidance: enumeration with the correct per-draw probability and binomial weights; distribution sums to 1; follow-up counts wrapped in B(n, p) with the complement taken before logs. This is authored classroom guidance.