Exercise walk-through

5.5
Successive impacts and wall rebounds

Pearson Edexcel · International A-Level · Further Mathematics

Centres of mass: particles, laminas and wire frameworks

- For particles of masses $m_1, m_2, \ldots$ at $x_1, x_2, \ldots$, the **centre of mass** is at
- and in the same way for $\bar{y}$.
- For a uniform composite body, use a table:
- 1. split the body into pieces whose centres of mass you know (the booklet gives arcs, sectors, semicircles and triangles); 2. use area as the "mass" of a lamina, and length as the "mass" of a wire; 3. take moments about a convenient line.

$$\bar{x} = \frac{\sum mx}{\sum m}$$

Suspending laminas and frameworks (2)

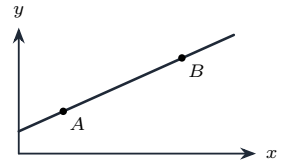
- $\tan\theta = \dfrac{1 - \frac{3}{3\pi+2}}{1 + \frac{3}{3\pi+2}} = \dfrac{3\pi - 1}{3\pi + 5}$, so $\theta = 30.3°$.
- With O as origin and OA along the x-axis: the rectangle has area 24 and centre $(3, 2)$; the square has area 4 and centre $(5, 3)$.
- $\overline{x} = \dfrac{24 \times 3 - 4 \times 5}{20} = 2.6$ cm and $\overline{y} = \dfrac{24 \times 2 - 4 \times 3}{20} = 1.8$ cm.
- OG is vertical, so the angle between OA and the vertical is $\tan^{-1}\frac{1.8}{2.6} = 34.7°$.

Collisions and restitution (2)

- Solving: $v_P = \dfrac{(2-e)u}{3}$ and $v_Q = \dfrac{2(1+e)u}{3}$.
- The **kinetic energy** after is $\frac{1}{2}(2m)v_P^2 + \frac{1}{2}mv_Q^2 = \dfrac{mu^2}{3}(2+e^2)$. Before, it was mu^2, so the loss is $\dfrac{mu^2}{3}(1-e^2)$.
- If the impulse on Q is $\frac{8}{9}mu$, then $m \times \dfrac{2(1+e)u}{3} = \dfrac{8mu}{9}$, so $e = \frac{1}{3}$, $v_Q = \frac{8}{9}u$ and $v_P = \frac{5}{9}u$.

Exercises 2.1·2.2

2.1 A ball hits a smooth wall at $8\ \mathrm{m\,s^{-1}}$ perpendicularly with $e = 0.4$. Find its rebound speed. [1]

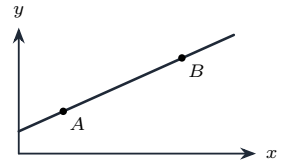


2.2 A 1 kg ball moving right at $6\ \mathrm{m\ s^{-1}}$ hits a wall ($e = \frac{1}{3}$) and rebounds into a stationary 2 kg ball, with which it collides with restitution $\frac{1}{2}$. Find the 2 kg ball's speed after. [5]



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2.1 $0.4 \times 8 = 3.2\ \mathrm{m\,s^{-1}}$.

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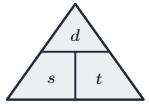


2.2 The ball rebounds at $6 \times \frac{1}{3} = 2\ \mathrm{m\,s^{-1}}$, leftward, so -2 with right positive. Momentum: $1(-2) = v_1 + 2v_2$; restitution with approach speed 2: the 2 kg ball, ahead on the left, separates leftward faster, so $\frac{1}{2} = \frac{v_1 - v_2}{2}$ and $v_1 = v_2 + 1$. Substituting: $-2 = (v_2+1)+2v_2 = 3v_2 + 1$, so $v_2 = -1$ and $v_1 = 0$: the 2 kg ball moves off at $\mathbf{1}\ \mathrm{m\,s^{-1}}$ to the left and the 1 kg ball stops.

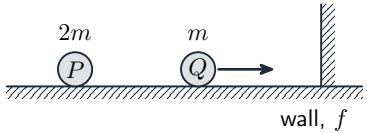
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Exercises 2.3·3.1

2.3 Three particles A, B, C in a line: A (m) at $3u$ hits B (m) at rest with $e = \frac{1}{2}$; B then hits C (2m) at rest with $e = \frac{1}{4}$. Find C's final speed. [6]

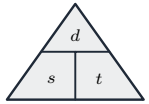


3.1 *(Edexcel IAL WME02 June 2025 Q5d.)* Following the collision of question 3.1 on the collisions sheet (masses $2m$ and m, restitution $e = \frac{1}{3}$): Q now moves towards a fixed vertical wall at right angles to its motion and rebounds with coefficient of restitution f. Given that immediately after the rebound Q has the same speed as P, find f. [3]



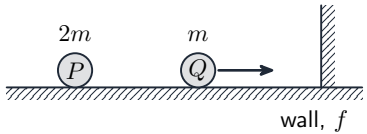
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2.3 First collision (A into B): $3u = v_A + v_B$ and $\frac{1}{2}(3u) = v_B - v_A$: $v_B = \frac{9}{4}u$, $v_A = \frac{3}{4}u$. Second (B into C): $\frac{9}{4}u = v'_B + 2v_C$ and $\frac{1}{4} \cdot \frac{9}{4}u = v_C - v'_B$: $v_C - v'_B = \frac{9u}{16}$. Adding: $3v_C = \frac{36u}{16} + \frac{9u}{16} = \frac{45u}{16}$, so $v_C = \frac{15u}{16}$.

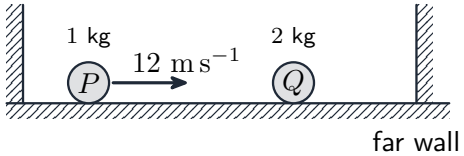
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3.1 After the first collision: $v_Q = \frac{2(1+\frac{1}{3})u}{3} = \frac{8}{9}u$ and $v_P = \frac{(2-\frac{1}{3})u}{3} = \frac{5}{9}u$. Rebound: $f \cdot \frac{8}{9}u = \frac{5}{9}u$, so $f = \frac{5}{8}$.

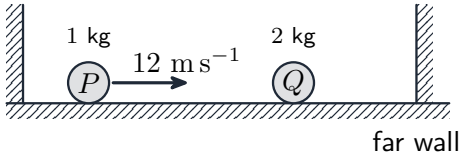
Exercise 3.2

3.2 *(Original.)* Two balls P (1 kg) and Q (2 kg) lie in a line on a smooth floor between two parallel walls. P is projected at 12 $\mathrm{m\,s^{-1}}$ towards Q; they collide with $e = \frac{1}{2}$; Q then hits the far wall with $e = \frac{1}{3}$ and rebounds to collide with P again. Find the speeds of both balls after the second P–Q collision, given it has restitution $\frac{1}{3}$. [8]



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3.2 First collision: $12 = v_P + 2v_Q$ and $\frac{1}{2}(12) = v_Q - v_P$ give $3v_Q = 18$: $v_Q = 6$, $v_P = 0$ (P stops). Q hits the far wall and rebounds at $6 \times \frac{1}{3} = 2$ leftward, towards the stationary P: approach speed 2. Right positive, momentum: $2(-2) = v'_P + 2v'_Q$, i.e. $-4 = v'_P + 2v'_Q$. Restitution: the separation speed after is $\frac{1}{3} \times 2 = \frac{2}{3}$ with P (ahead on the left) separating leftward faster, so $v'_Q - v'_P = \frac{2}{3}$, i.e. $v'_P = v'_Q - \frac{2}{3}$. Substituting: $-4 = 3v'_Q - \frac{2}{3}$, so $v'_Q = -\frac{10}{9}$ and $v'_P = -\frac{16}{9}$. Both move leftward: P at $\mathbf{\frac{16}{9}} = 1.78$ $\mathrm{m\,s^{-1}}$, Q at $\mathbf{\frac{10}{9}} = 1.11$ $\mathrm{m\,s^{-1}}$.

Local review guidance: wall rebounds as pure speed scaling before any momentum work; each collision solved completely with a signed diagram; “same speed” conditions applied to the correct stage. This is authored classroom guidance.

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