Exercise walk-through

2.2
The method of differences

Pearson Edexcel · International A-Level · Further Mathematics

Further Pure Mathematics 2: inequalities, further complex numbers and differential equations

- solve inequalities involving the modulus sign, and sum series by the **method of differences**;
- use **Euler's relation** and **de Moivre's theorem**, and find the nth roots of complex numbers;
- sketch loci in the Argand diagram, and find images under transformations $w = \frac{az+b}{cz+d}$;

The method of differences

- The **method of differences** sums a series by writing each term as a difference, usually using **partial fractions**, so that most of the terms cancel.
- Write out the terms for $r = 1$, $r = 2$, ..., $r = n - 1$ and $r = n$ to show clearly what cancels; this earns marks.
- **Worked example 3.** Show that $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \frac{n}{2(n+2)}$.
- Check with $n = 1$: both sides are $\frac{1}{6}$.

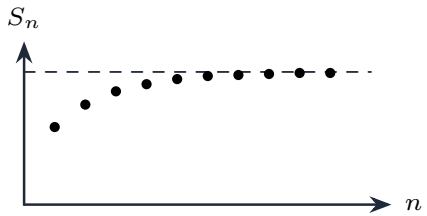
$$\frac{1}{(r+1)(r+2)} = \frac{1}{r+1} - \frac{1}{r+2}$$

Check yourself

- 1.
- Solve $|2x-1| < |x+4|$. 2.
- Use the **method of differences** to find $\sum_{r=1}^{n} \frac{1}{r(r+1)}$. 3.
- Write $\left(\sqrt{3}+i\right)^6$ in the form $a+bi$. 4.

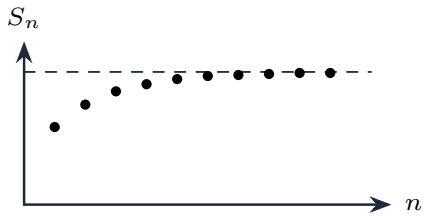
Exercise 2.1

2.1 Show that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [2]



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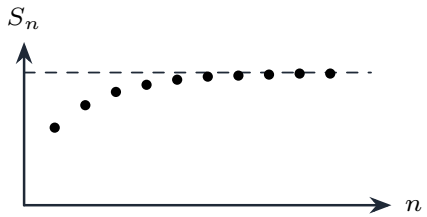
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2.1 $\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$. The sum telescopes to $1 - \frac{1}{n+1} = \frac{n}{n+1}$.

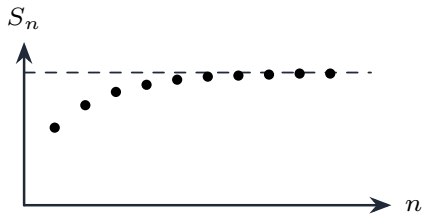
Exercise 2.2

2.2 Show that $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$. [4]



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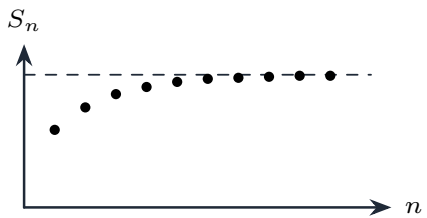


2.2 $\dfrac{1}{(2r-1)(2r+1)} = \dfrac{1}{2}\left[\dfrac{1}{2r-1} - \dfrac{1}{2r+1}\right]$ (check: $\frac{1}{2} \cdot \frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)} = \frac{1}{(2r-1)(2r+1)}$ ✓).

The sum is $\frac{1}{2}\left(1 - \dfrac{1}{2n+1}\right) = \frac{1}{2} \cdot \dfrac{2n}{2n+1} = \dfrac{n}{2n+1}$.

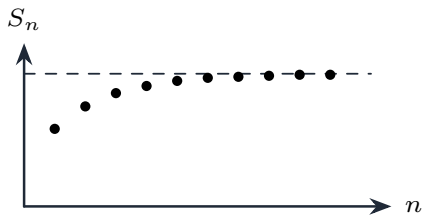
Exercise 2.3

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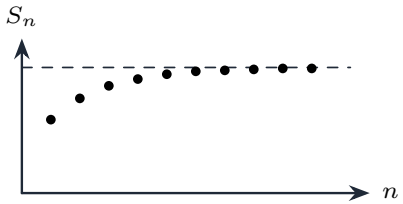


2.3 $r \cdot r! = (r+1-1)r! = (r+1)! - r!$. The sum telescopes to $(n+1)! - 1! = (n+1)! - 1$. (Check $n = 2$: $1 + 2 \cdot 2 = 5 = 6 - 1$ ✓.)

Exercise 3.1

3.1 *(Original.)*

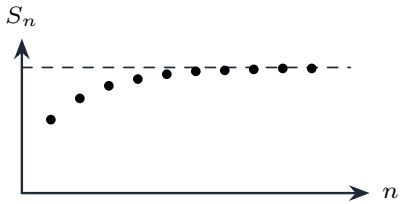
(a) Show that $\dfrac{1}{r(r+1)(r+2)} = \dfrac{1}{2}\left[\dfrac{1}{r(r+1)} - \dfrac{1}{(r+1)(r+2)}\right]$. (b) Hence find, in its simplest form, $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$. [6]



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3.1 (a) $\dfrac{1}{2}\left[\dfrac{(r+2)-r}{r(r+1)(r+2)}\right] = \dfrac{1}{r(r+1)(r+2)}$ ✓. (b) The sum is $\dfrac{1}{2}\left[\dfrac{1}{1\cdot 2} - \dfrac{1}{(n+1)(n+2)}\right]$ $\dfrac{1}{4} - \dfrac{1}{2(n+1)(n+2)}$. (Check $n=1$: $\frac{1}{6}$ and $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ ✓.)

Exercise 3.2

3.2 *(Adapted from Edexcel IAL WFM02 June 2025 Q5.)* (a) Express $\dfrac{1}{(r+1)(r+2)}$ in partial fractions. (b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \frac{n}{2(n+2)}$. (c) Determine the smallest value of n for which the sum exceeds 0.49. [10]

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3.2 (a) $\dfrac{1}{(r+1)(r+2)} = \dfrac{1}{r+1} - \dfrac{1}{r+2}$. (b) Writing out $r = 1$ to n: $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{1}{2} - \frac{1}{n+2} = \frac{n}{2(n+2)}$. (c) $\dfrac{n}{2(n+2)} > 0.49 \Rightarrow n > 0.98(n+2) \Rightarrow 0.02n > 1.96 \Rightarrow n > 98$, so the smallest is $n = 99$ (check: $\frac{99}{202} \approx 0.4901 > 0.49$ ✓; $\frac{98}{200} = 0.49$ not strictly greater ✓).

Local review guidance: correct partial-fraction or factorial split verified; the cancellation displayed through first/last terms; closed form simplified and spot-checked. This is authored classroom guidance.