Course notes

Pearson Edexcel · International A-Level · Further Mathematics

YFM01: course teaching notes

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These are original teaching notes, and each lesson is labelled with its tier.

FP1 · Complex arithmetic and quadratic roots

What if the root is not real?

- The equation $x^2+1=0$ has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

The method

- Write $z=a+bi$ with $i^2=-1$.
- Add components, multiply brackets and use a conjugate to make a division denominator real.
- Non-real roots of a real-coefficient quadratic occur in conjugate pairs.

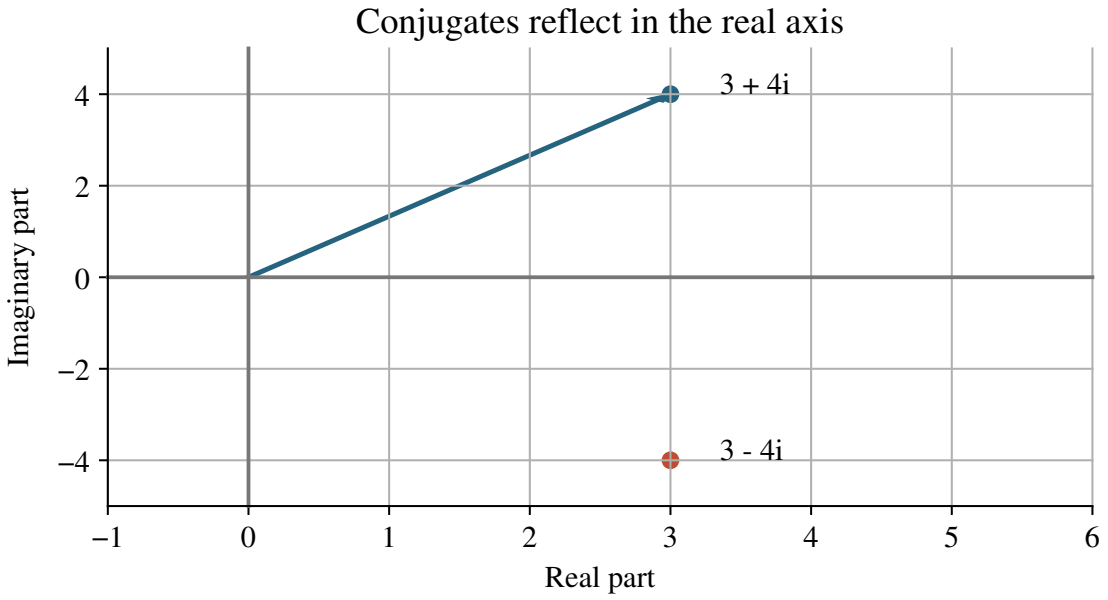
A worked case

$$z = a + bi, \qquad |z| = \sqrt{a^2 + b^2}, \qquad z\overline{z} = a^2 + b^2$$

■ Worked example

For $z=3+4i$, $|z|=5$ and z conjugate=3-4i. The reciprocal is $(3-4i)/25$. For $x^2-6x+13=0$, the roots are $3\pm2i$. Multiplying by the conjugate makes a complex denominator real.

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Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

> ■ **Watch out**
>
> **Not true:** For a+bi, its modulus is a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

Applying it

- FP1 uses complex arithmetic, modulus, Argand representation and quadratic roots.
- De Moivre powers and roots of unity are reserved for FP2.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with i^2=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

FP1 · Parabolas and rectangular hyperbolas

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parabola** 抛物线: The locus of points equally distant from a fixed focus and a fixed directrix.

The method

- For y^2=4ax, the focus is (a,0), directrix x=-a and vertex (0,0).
- For xy=c^2, the coordinate axes are asymptotes.
- Read the parameter from the coefficient rather than assuming it equals 4a.

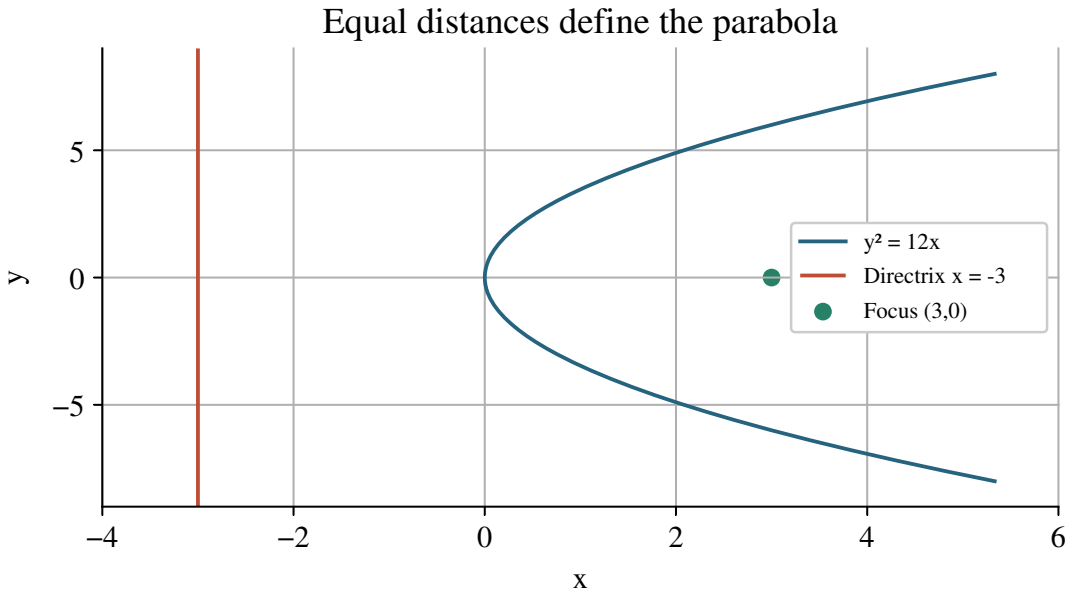
A worked case

$$y^2 = 12x, \quad xy = 9$$

■ **Worked example**

For y^2=12x, a=3: the focus is (3,0) and directrix x=-3. At x=3, y=±6; at (3,6), distance to the focus and directrix is 6. For xy=9, the point (3,3) lies on the hyperbola and x=0,y=0 are its asymptotes.

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Where it goes wrong

- The coefficient of x is 4a, not a.
- Include both signs when solving y^2.
- An asymptote is approached; it is not a finite intercept of $xy=c^2$.

> **Watch out**
>
> **Not true:** For $y^2=4ax$, the focus x-coordinate is 4a. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

Applying it

- FP1 coordinate-geometry support for parabolas and rectangular hyperbolas.
- Ellipse area and ellipse parametrisation are excluded.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP1.
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■ **Key idea**

- **Parabola**: The locus of points equally distant from a fixed focus and a fixed directrix.
- For $y^2=4ax$, the focus is (a,0), directrix x=-a and vertex (0,0).
- Watch for: the coefficient of x is 4a, not a.

FP1 · Matrix operations, inverses and plane transformations

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

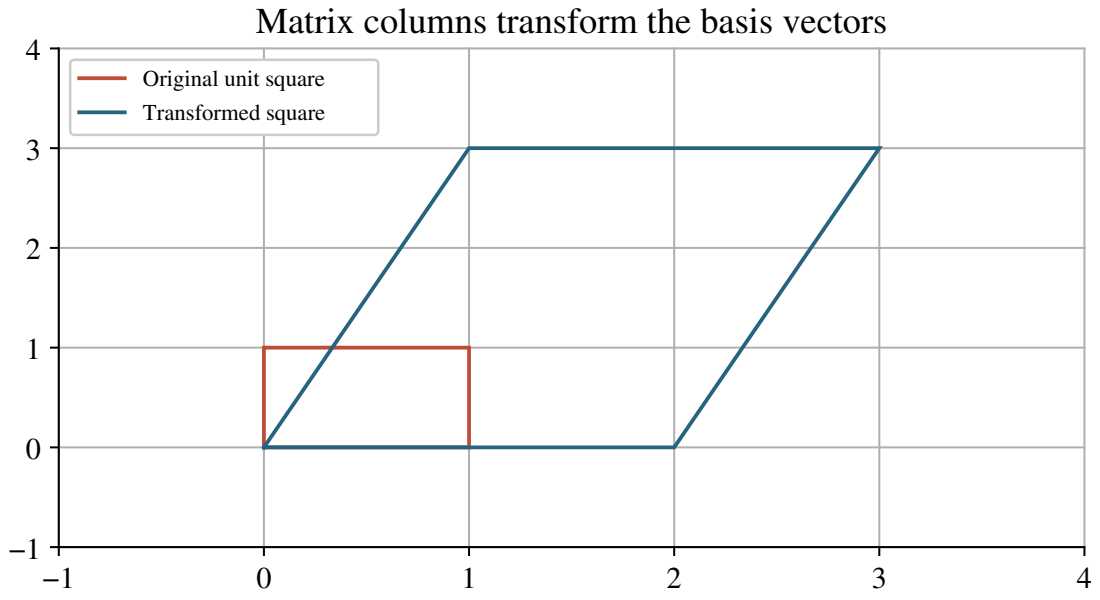
A worked case

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ **Worked example**

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. Check by multiplying A by its inverse to obtain the identity matrix.

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Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

■ Watch out

Not true: Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- FP1 uses 2×2 matrix operations, inverses and plane transformations.
- Eigenvalues and 3×3 diagonalisation belong to FP3, not this unit.

In your course

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■ **Key idea**

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

FP1 · Bisection and Newton root finding

How reliable is an approximation?

- A nonlinear equation has no convenient exact solution. An approximation still needs evidence that it is reliable.
- This lesson studies **iteration** 迭代: A repeated update in which each new approximation is calculated from the previous one.

The method

- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Newton updates x to x-f(x)/f prime(x), provided the derivative is nonzero.

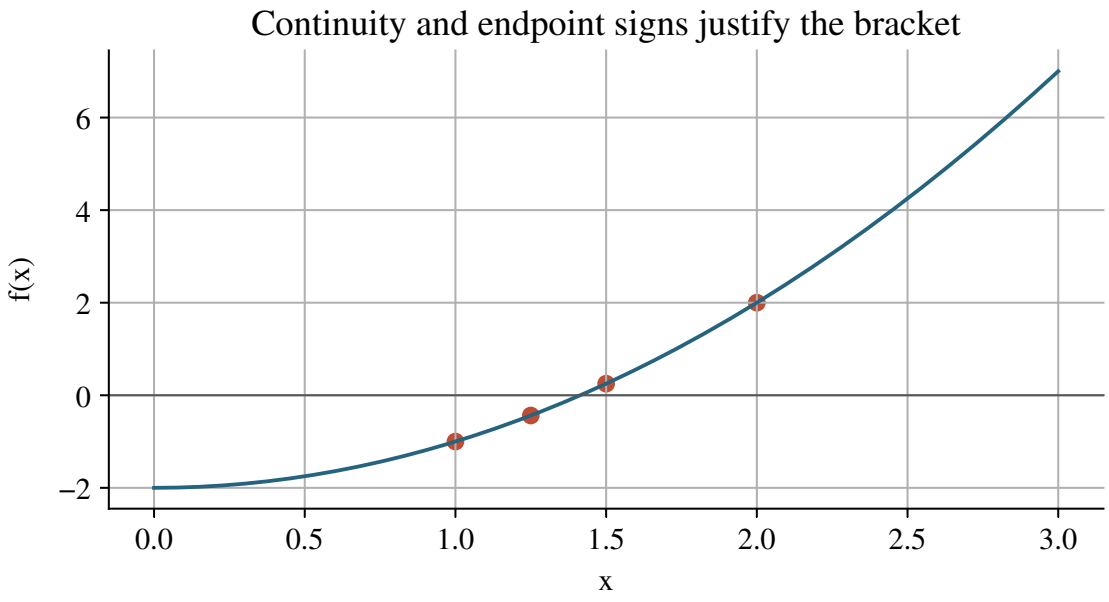
A worked case

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

■ **Worked example**

For f(x)=x^2-2, f(1)=-1 and f(2)=2. At midpoint 1.5, f=0.25, so the new bracket is [1,1.5]. At midpoint 1.25, f=-0.4375, so the next bracket is [1.25,1.5]. Newton from 1.5 gives 1.4166667.

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Where it goes wrong

- A sign change must occur on an interval where the function is continuous.
- Newton can fail if its derivative is zero, or if its iterates leave the useful domain.

> ■ **Watch out**
>
> **Not true:** Every sign change proves a root, including one across a discontinuity. A sign change shows a root only if the function is continuous there: 1/x changes sign across x=0 but has no root.

Applying it

- FP1 numerical methods concern roots: bisection, interpolation and Newton methods.
- Numerical integration is not included in this lesson.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Iteration**: A repeated update in which each new approximation is calculated from the previous one.
- For a continuous function with a sign change, bisect a root bracket and keep the half with opposite endpoint signs.
- Watch for: A sign change must occur on an interval where the function is continuous.

FP1 · Finite sums and induction

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **sigma notation** 求和符号: Notation that adds indexed terms over a stated finite range.

The method

- For k=1 to n, sum k=n(n+1)/2, sum k^2=n(n+1)(2n+1)/6 and sum k^3=[n(n+1)/2]^2.
- Differences between consecutive terms can telescope.
- In induction, verify the starting case and show the next case follows from the assumed case.

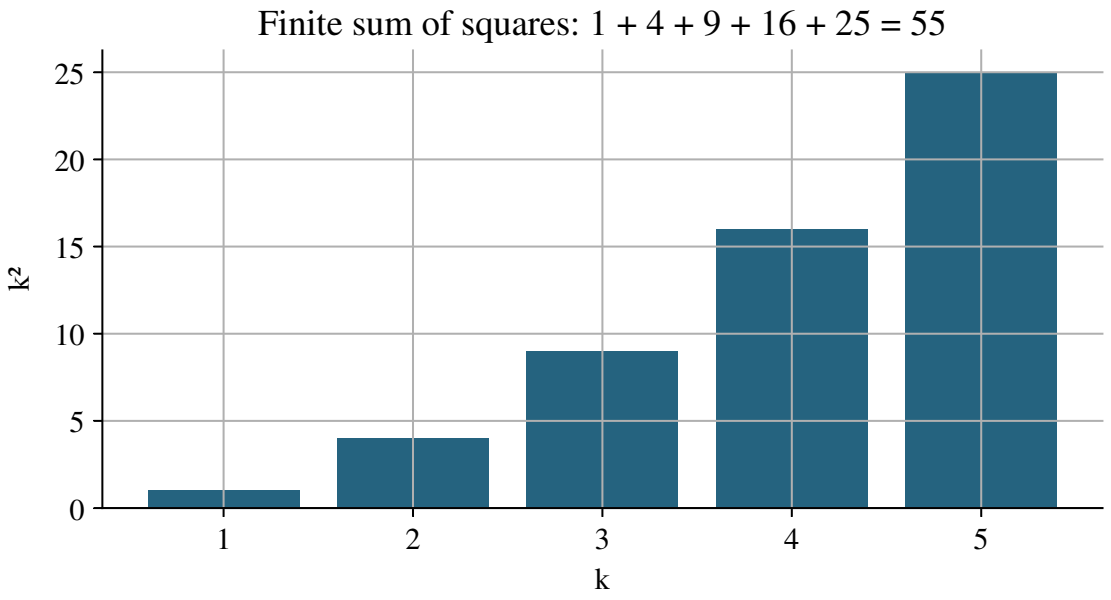
A worked case

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

■ **Worked example**

For n=5, sum k=15, sum k^2=1+4+9+16+25=55 and sum k^3=1+8+27+64+125=225. In proving a sum formula, adding the (n+1)th term to the assumed expression must produce the formula with n replaced by n+1.

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Where it goes wrong

- The sum of squares is not the square of the sum.
- State the index range and check the first case; an inductive step by itself is not a complete induction proof.

> ■ **Watch out**
>
> **Not true:** The sum of squared terms equals the square of their sum. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

Applying it

- FP1 finite series and induction support only.
- Maclaurin and Taylor approximation belong to later further-pure units and are excluded.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ **Key idea**

- **Sigma notation**: Notation that adds indexed terms over a stated finite range.
- For k=1 to n, sum k=n(n+1)/2, sum k^2=n(n+1)(2n+1)/6 and sum k^3=[n(n+1)/2]^2.
- Watch for: the sum of squares is not the square of the sum.

FP1 · Deduction, contradiction and induction

When does a pattern become a proof?

- Checking several integers can suggest a pattern. What turns the pattern into a proof for every integer?
- This lesson studies **counterexample** 反例: A single valid case that disproves a universal claim.

The method

- A deductive proof starts from stated definitions or assumptions.
- A counterexample refutes an all-values claim.
- Induction requires a base case and an implication from n=k to n=k+1; checking successive cases alone is not induction.

A worked case

$$(2a+1)+(2b+1)=2(a+b+1)$$

■ **Worked example**

For odd integers 2a+1 and 2b+1, their sum is 2(a+b+1), hence even. The claim n^2+n+41 is always prime fails at n=41: the value is 41×43=1763. For 1+...+n=n(n+1)/2, the induction step adds k+1 to the assumed sum.

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Odd + odd: (2a + 1) + (2b + 1)

Factor 2: 2(a + b + 1)

The bracket is an integer ⇒ the sum is even

Where it goes wrong

- Do not assume the conclusion while proving it.
- In contradiction, identify the impossible consequence and reject the initial contrary assumption.
- Induction is not compulsory in AQA 7357; it belongs in its own appropriate further-mathematics scope.

> ■ **Watch out**
>
> **Not true:** Checking the first ten integers proves a claim for every positive integer. A pattern can hold for the first ten cases and then fail; a proof must cover every case.

Applying it

- AQA proof includes deduction, exhaustion and contradiction.
- IAL P2 introduces exhaustion and counterexample, while P4 introduces contradiction.
- Keep the method matched to the named unit.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Counterexample**: A single valid case that disproves a universal claim.
- A deductive proof starts from stated definitions or assumptions.
- Watch for: do not assume the conclusion while proving it.

FP2 · Quadratics and inequalities

Which widths make enough space?

- A rectangular enclosure has area x(10-x). What widths give at least 21 square metres?
- This lesson studies **discriminant** 判别式: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.

The method

- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

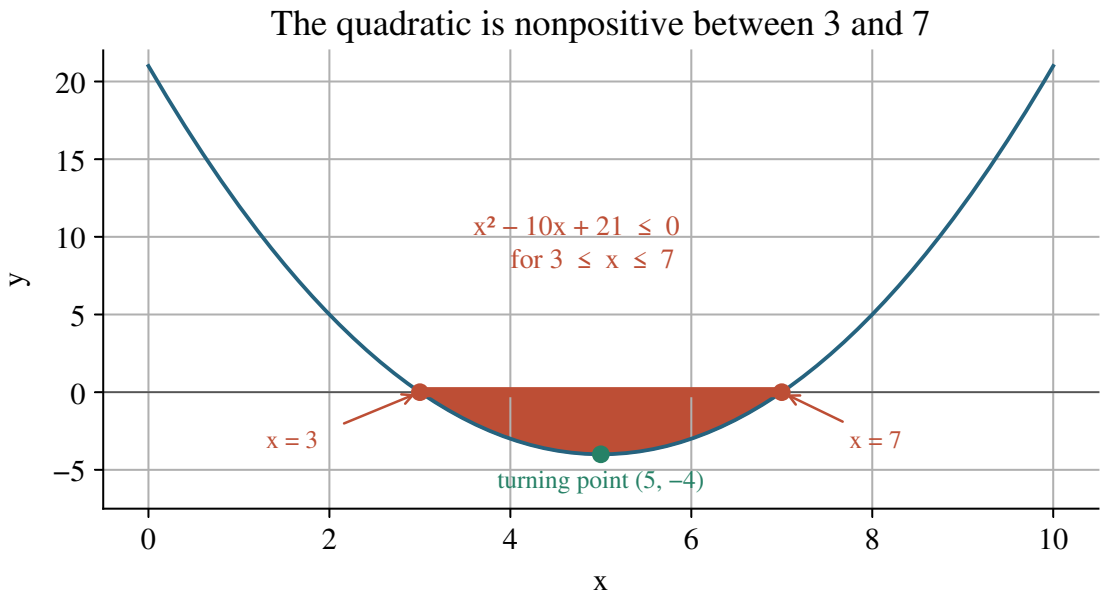
A worked case

$$ax^2 + bx + c = 0, \qquad \Delta = b^2 - 4ac$$

■ Worked example

The condition is x(10-x)≥21, so x^2-10x+21≤0. Factor (x-3)(x-7)≤0. The upward parabola is nonpositive between its roots, giving 3≤x≤7. The maximum area is 25 at x=5.

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Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

> **Watch out**
>
> **Not true:** A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- Use the vertex to interpret an optimum.
- Check an endpoint and a point between the roots; the algebra and graph should tell the same story.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Discriminant**: The quantity b^2-4ac that determines the real roots of ax^2+bx+c=0.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

FP2 · Series, finite differences and Taylor expansions

Can a polynomial replace an exponential nearby?

- Near a known point, a complicated function can be replaced by a polynomial approximation. The approximation needs a centre and a valid scale.
- This lesson studies **Taylor polynomial** 泰勒多项式: A polynomial formed from a function's derivatives at a chosen centre.

The method

- About x=a, f(x)=f(a)+f prime(a)(x-a)+f double prime(a)$(x-a)^2/2!$+... when the expansion is valid.
- At a=0 it is a Maclaurin expansion.
- Finite sums may also simplify by cancellation or standard sum formulae.

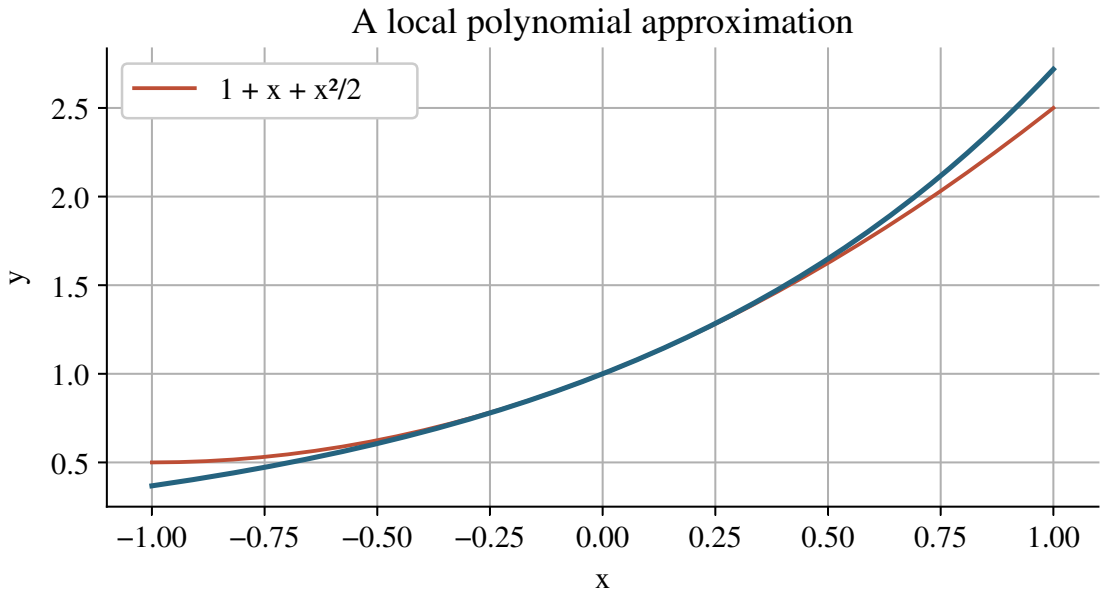
A worked case

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

■ **Worked example**

The first three terms of e^x are $1+x+x^2/2$. At x=0.1 this gives 1.105, close to e^0.1≈1.105170. For the sum of k^2 from k=1 to 5, n(n+1)(2n+1)/6 gives 5×6×11/6=55.

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Where it goes wrong

- The factorial belongs in the denominator of every Taylor coefficient.
- The expansion centre is not always zero.
- An approximation close to its centre can be poor far away.

> ■ **Watch out**
>
> **Not true:** A Taylor polynomial is exact for the original function at every input. A Taylor polynomial is an approximation; it is usually accurate only near its centre.

Applying it

- For a telescoping series, write several terms and identify both surviving ends.
- For a Taylor approximation, state its order and compare with a known value or a remainder estimate where required.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Taylor polynomial**: A polynomial formed from a function's derivatives at a chosen centre.
- About x=a, f(x)=f(a)+f prime(a)(x-a)+f double prime(a)(x-a)^2/2!+... when the expansion is valid.
- Watch for: the factorial belongs in the denominator of every Taylor coefficient.

FP2 · Complex numbers and roots

What if the root is not real?

- The equation x^2+1=0 has no real roots. Extending the number system lets us represent and calculate with both roots.
- This lesson studies **complex conjugate** 共轭复数: The number obtained by reversing the sign of the imaginary part.

The method

- Write z=a+bi with i^2=-1.
- The conjugate is a-bi and z times its conjugate=a^2+b^2.
- The modulus is √(a^2+b^2); the argument needs the correct quadrant.
- Conjugate roots occur for polynomials with real coefficients.

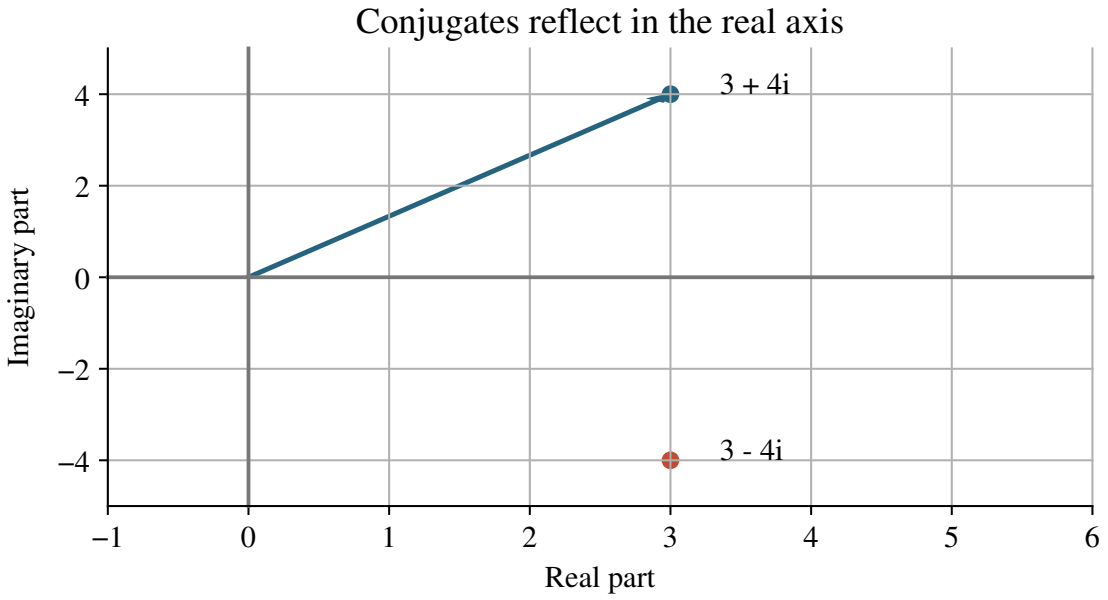
A worked case

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■ **Worked example**

For z=3+4i, |z|=5 and z conjugate=3-4i. The reciprocal is (3-4i)/25. For x^2-6x+13=0, the roots are 3±2i. Multiplying by the conjugate makes a complex denominator real.

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Where it goes wrong

- The square root of a sum is not the sum of the square roots.
- An inverse tangent alone may return the wrong quadrant.
- Do not confuse modulus with squared modulus.

■ Watch out

Not true: For a+bi, its modulus is a+b. The modulus is $\sqrt{(a^2+b^2)}$: for 3+4i it is 5, not 7.

Applying it

- Draw the real axis horizontally and imaginary axis vertically.
- For products, moduli multiply and arguments add; powers extend this geometric pattern through de Moivre's theorem.

In your course

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■ **Key idea**

- **Complex conjugate**: The number obtained by reversing the sign of the imaginary part.
- Write z=a+bi with i^2=-1.
- Watch for: the square root of a sum is not the sum of the square roots.

FP2 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

The method

- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Use an initial condition to find A.
- Euler's method takes y next=y+h f(x,y), with a chosen step h.

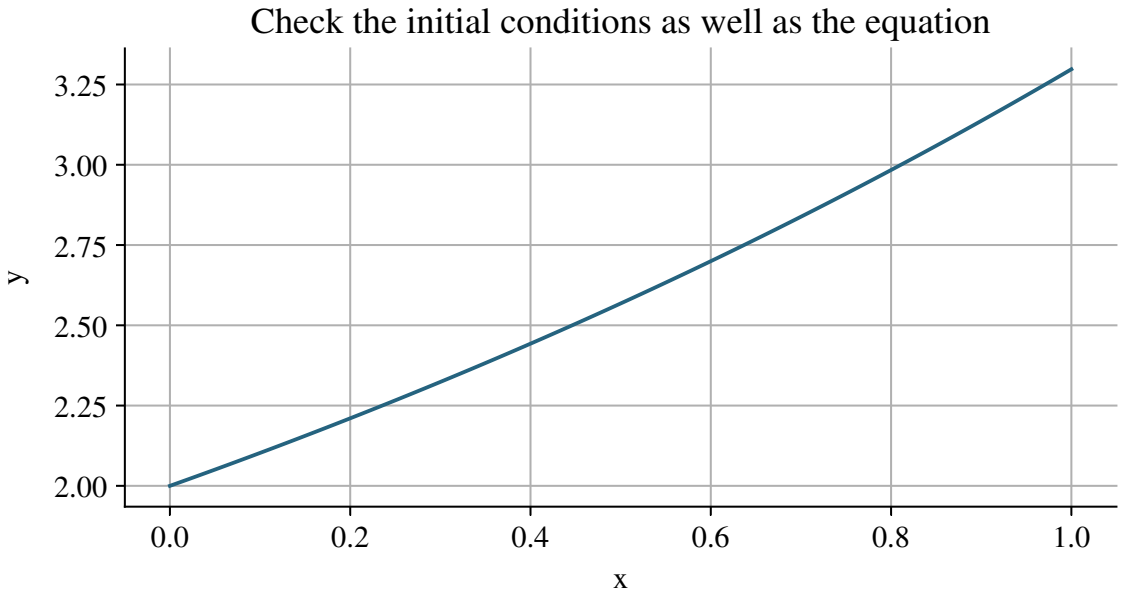
A worked case

$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

■ **Worked example**

With dy/dx=0.5y and y(0)=2, y=2e^(0.5x). Euler with h=0.2 gives y(0.2)≈2+0.2×1=2.2 and y(0.4)≈2.2+0.2×1.1=2.42. The exact second value is about 2.4428.

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Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** Every step size gives exactly the same Euler approximation. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions.
- This is Further Pure scope; check the named unit.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP2.
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■ **Key idea**

- **Initial condition**: A specified value that selects a particular solution from a family.
- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

FP2 · Second-order linear differential equations

Why do we need two initial conditions?

- A displacement model involves both velocity and acceleration. Solving only a first-order rate equation cannot capture both initial conditions.
- This lesson studies **complementary function** 互补函数: The general solution of the associated homogeneous linear differential equation.

The method

- For y double prime+ay prime+by=f(x), solve the auxiliary quadratic for the homogeneous part.
- Add a suitable particular integral for the forcing term.
- Repeated and complex roots require their correct forms.

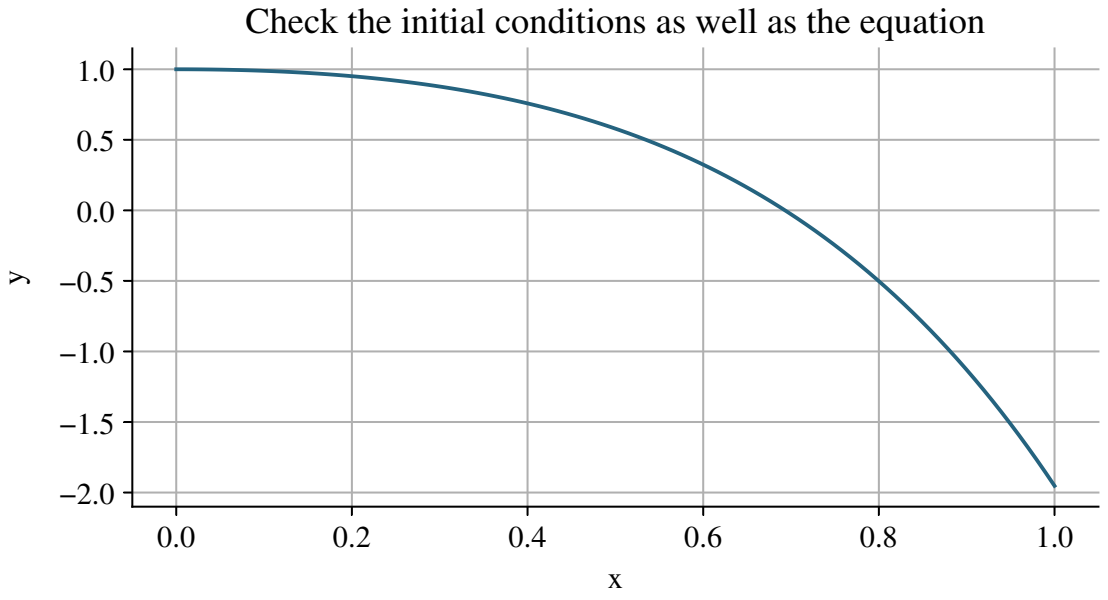
A worked case

$$y'' + ay' + by = f(x), \qquad m^2 + am + b = 0$$

■ **Worked example**

For y double prime-3y prime+2y=0, the auxiliary equation is m^2-3m+2=0 with roots 1,2. Hence y=Ae^x+Be^(2x). With y(0)=1 and y prime(0)=0, A+B=1 and A+2B=0, so A=2,B=-1.

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Where it goes wrong

- Two arbitrary constants need two independent conditions.
- If the trial particular integral duplicates a complementary-function term, multiply the trial by x as required.
- Do not discard a valid oscillatory solution because the auxiliary roots are complex.

> ■ **Watch out**
>
> **Not true:** One initial value is sufficient to determine both constants in a second-order general solution. A second-order solution has two constants, so it needs two conditions.

Applying it

- Differentiate the final expression and substitute into the original differential equation.
- Check both initial conditions separately, and interpret the permitted solution interval.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Complementary function**: The general solution of the associated homogeneous linear differential equation.
- For y double prime+ay prime+by=f(x), solve the auxiliary quadratic for the homogeneous part.
- Watch for: two arbitrary constants need two independent conditions.

FP2 · Polar curves and area

Why describe a curve by direction?

- A petal-shaped curve is simpler when its radius depends on direction. Cartesian coordinates can hide this structure.
- This lesson studies **polar coordinate** 极坐标: A position described by distance r and angle θ from a chosen origin and reference ray.

The method

- Use x=r cosθ and y=r sinθ.
- A polar area is one half the integral of r^2 with respect to θ over a correctly chosen interval.
- Identify symmetry and repeated tracing before selecting limits.

A worked case

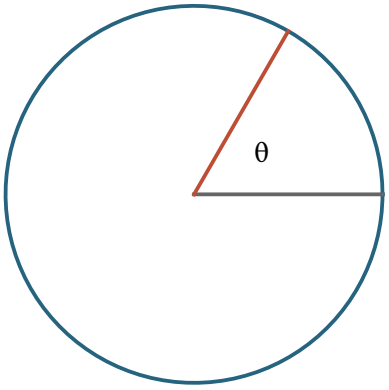
$$x = r\cos\theta, \quad y = r\sin\theta, \quad A = \frac{1}{2}\int r^2\,d\theta$$

■ Worked example

For r=2 and $\theta=\pi/3$, x=1 and y=$\sqrt{3}$. For the circle r=2 over a complete turn, A=(1/2) integral_0^(2π) 4 dθ=4π. A half-turn gives 2π, exactly half the disk.

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A radius fixes distance from the centre



radius × angle gives arc length

Where it goes wrong

- Negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.
- A parametrisation may trace the same region more than once, so a full parameter interval can overcount area.

■ Watch out

Not true: Using any full parameter interval counts each polar region exactly once. The interval must match the curve's period; too long an interval counts a region twice.

Applying it

- Sketch enough points to establish orientation and bounds.
- For an enclosed region between two curves, determine intersections and which radial square contributes the outer boundary on each interval.

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In your course

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- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Polar coordinate**: A position described by distance r and angle θ from a chosen origin and reference ray.
- Use x=r cosθ and y=r sinθ.
- Watch for: negative r places a point in the opposite direction; it is not an ordinary negative distance along the same ray.

FP3 · Hyperbolic functions and inverse relations

How can growth and decay make a symmetric curve?

- A hanging cable has a curved profile related to exponentials. Hyperbolic functions combine growth and decay symmetrically.
- This lesson studies **hyperbolic cosine** 双曲余弦: The function cosh x=(e^x+e^(-x))/2.

The method

- Define sinh x=(e^x-e^(-x))/2 and cosh x=(e^x+e^(-x))/2.
- Their identity is $\cosh^2$x-$\sinh^2$x=1.
- Derivatives are sinh prime=cosh and cosh prime=sinh.

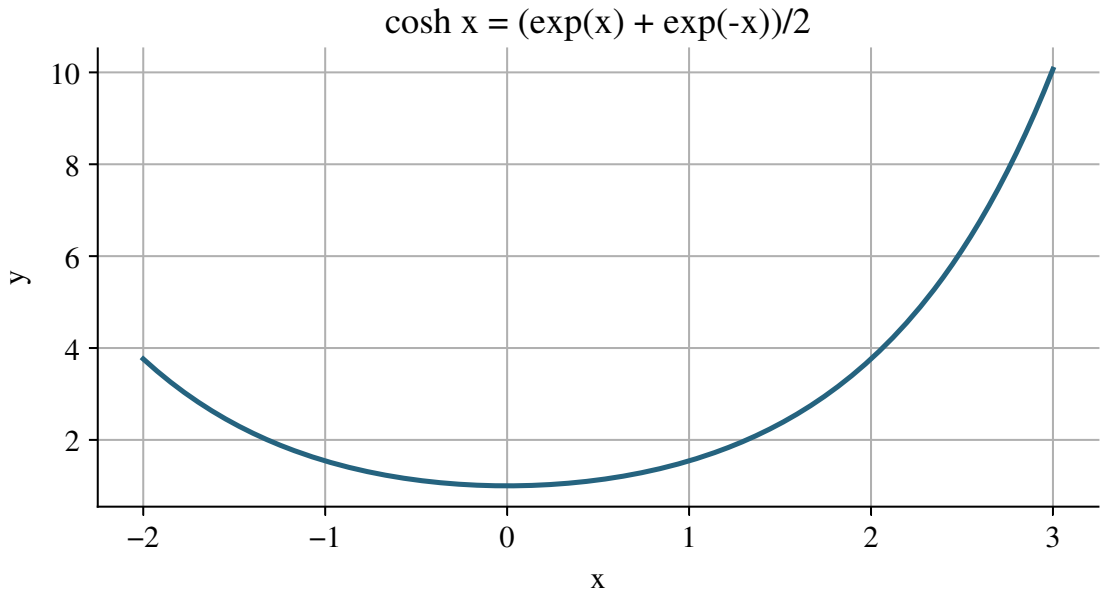
A worked case

$$\cosh x = \frac{e^{x} + e^{-x}}{2}, \qquad \cosh^{2} x - \sinh^{2} x = 1$$

Worked example

At x=ln2, e^x=2 and e^(-x)=1/2. Thus cosh x=1.25 and sinh x=0.75. Their squared difference is 1.5625-0.5625=1. To invert y=sinh x, solve a quadratic in e^x and choose the positive root.

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Where it goes wrong

- The hyperbolic identity has a minus sign. cosh is not one-to-one on all real inputs; its usual inverse uses $x \geq 0$.
- Ordinary circular-trigonometric identities cannot be substituted unchanged.

> ■ **Watch out**
>
> **Not true:** The identity for hyperbolic functions is $\cosh^2 x + \sinh^2 x = 1$. The hyperbolic identity is $\cosh^2 x - \sinh^2 x = 1$.

Applying it

- Use exponential definitions to prove identities and solve equations.
- State domain restrictions for inverse functions before differentiating or integrating them.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Hyperbolic cosine**: The function cosh x=(e^x+e^(-x))/2.
- Define sinh x=(e^x-e^(-x))/2 and cosh x=(e^x+e^(-x))/2.
- Watch for: the hyperbolic identity has a minus sign. cosh is not one-to-one on all real inputs; its usual inverse uses x≥0.

FP3 · Conics, parametric curves and tangent reasoning

How can a rotating parameter trace an ellipse?

- An ellipse can be traced by a rotating parameter even when it is awkward to describe as a single y=f(x).
- This lesson studies **parametric equation** 参数方程: A representation in which coordinates are expressed using a common parameter.

The method

- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- For parametric curves, dy/dx=(dy/dt)/(dx/dt), where dx/dt≠0.
- A zero denominator may signal a vertical tangent.

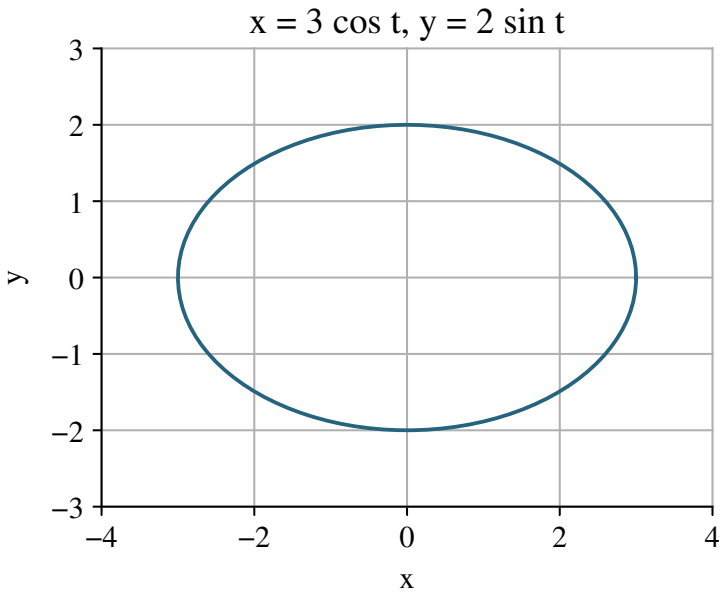
A worked case

$$x = a\cos t, \qquad y = b\sin t, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

■ **Worked example**

For x=3cos t,y=2sin t at t=π/4, the gradient is (2cos t)/(-3sin t)=-2/3. The point is (3/√2,√2). The area inside the ellipse is πab=6π; its semiaxes are 3 and 2.

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Where it goes wrong

- Eliminating a parameter may lose a domain restriction or direction of travel.
- A vertical tangent cannot be assigned a finite dy/dx.
- Distinguish semiaxes from full widths.

> ■ **Watch out**
>
> **Not true:** Eliminating a parameter preserves every restriction and direction automatically. Elimination can lose restrictions: $x=t^2$, $y=t$ gives $x=y^2$, but $x\geq 0$ and the direction of motion must be stated separately.

Applying it

- For advanced coordinate geometry, use the defining equation and a consistent parameter.
- For IAL P4 parametric integration, use the specification's restricted requirements rather than importing all Further Pure conics.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Parametric equation**: A representation in which coordinates are expressed using a common parameter.
- For x=a cos t,y=b sin t, eliminating t gives $x^2/a^2+y^2/b^2=1$.
- Watch for: eliminating a parameter may lose a domain restriction or direction of travel.

FP3 · Chain, product, quotient and implicit differentiation

Does the inside expression change too?

- A cost curve is a power of a changing expression. Differentiating the outer power alone misses the rate of its input.
- This lesson studies **chain rule** 链式法则: The rule that multiplies the outer derivative by the inner derivative for a composite function.

The method

- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- For uv, differentiate to u prime v+uv prime.
- For u/v, use (u prime v-uv prime)/v^2, where $v\neq0$.

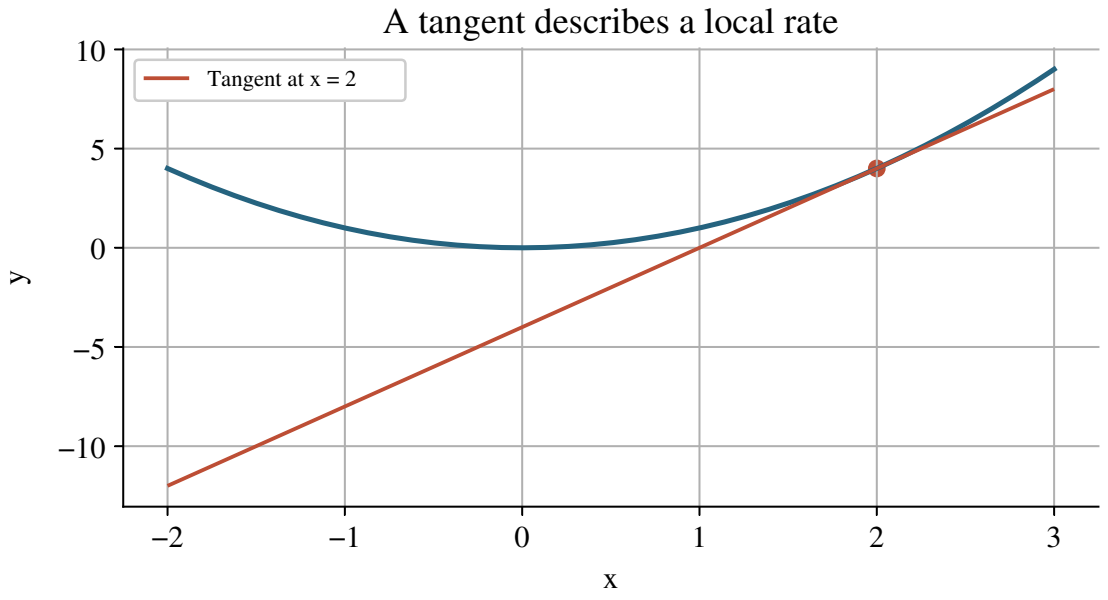
A worked case

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

■ Worked example

For y=(3x+1)^4, y prime=4(3x+1)^3×3=12(3x+1)^3. At x=0, the gradient is 12. For $x^2+y^2=25$, differentiate implicitly: 2x+2y y prime=0, so y prime=-x/y when $y\neq0$.

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Where it goes wrong

- A derivative of a product is not the product of the derivatives.
- In implicit differentiation, every differentiated function of y brings a dy/dx factor.
- A quotient's denominator is squared.

■ Watch out

Not true: The derivative of u(x)v(x) is u prime times v prime. The product rule is $(uv)'=u'v+uv'$: for $x \cdot x$ the derivative is 2x, not $1\times1=1$.

Applying it

- Choose a useful form before differentiating: expanding a short polynomial may be simpler.
- For related rates, write the relation in symbols, differentiate with respect to time, then substitute measured values.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Chain rule**: The rule that multiplies the outer derivative by the inner derivative for a composite function.
- For y=f(g(x)), y prime=f prime(g(x))g prime(x).
- Watch for: A derivative of a product is not the product of the derivatives.

FP3 · Substitution, parts and partial fractions

Can we integrate the two factors separately?

- A simple-looking product such as xe^x cannot be integrated by separately integrating each factor.
- This lesson studies **integration by parts** 分部积分法: An integration method based on the derivative of a product.

The method

- Use substitution when an inner derivative appears as a factor.
- By parts, integral u v prime =uv-integral u prime v.
- For a rational function, divide first if needed, then decompose into partial fractions.

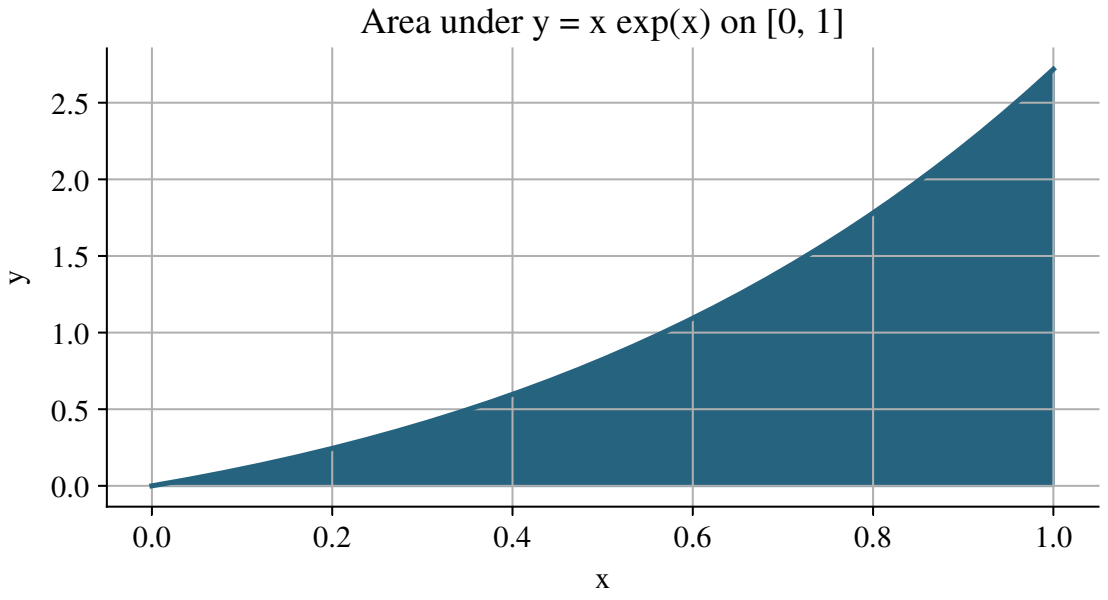
A worked case

$$\int uv'\,dx = uv - \int u'v\,dx$$

■ **Worked example**

For integral xe^x dx, take u=x and v prime=e^x. Then the integral is xe^x-e^x+C. For integral $2x/(x^2+1)$ dx, substitute $w=x^2+1$, dw=2x dx; the answer is $\ln(x^2+1)+C$.

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Where it goes wrong

- Choosing u and v prime well matters: the remaining integral should become simpler.
- In a definite substitution, either change the limits or return to x before applying the original limits.

> ■ **Watch out**
>
> **Not true:** The integral of a product equals the product of its separate integrals. A product needs a method such as integration by parts or substitution: $\int x \cdot x\,dx = x^3/3$, not $(x^2/2)(x^2/2)$.

Applying it

- Check by differentiating.
- For volumes of revolution around the x-axis use $V = \pi \int y^2\,dx$; do not confuse the square of a function with the integral of the function.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Integration by parts**: An integration method based on the derivative of a product.
- Use substitution when an inner derivative appears as a factor.
- Watch for: choosing u and v prime well matters: the remaining integral should become simpler.

FP3 · Spatial vectors, lines and angles

Can nonparallel paths still miss each other?

- Two paths in space can be neither parallel nor intersecting. A two-dimensional sketch may conceal their separation.
- This lesson studies **direction vector** 方向向量: A nonzero vector giving the direction of a line.

The method

- A line has r=a+λb with position vector a and nonzero direction b.
- To find an intersection, equate all components and solve for both line parameters.
- For an angle use a · b=|a||b|cosθ where scalar products are in the named syllabus.

A worked case

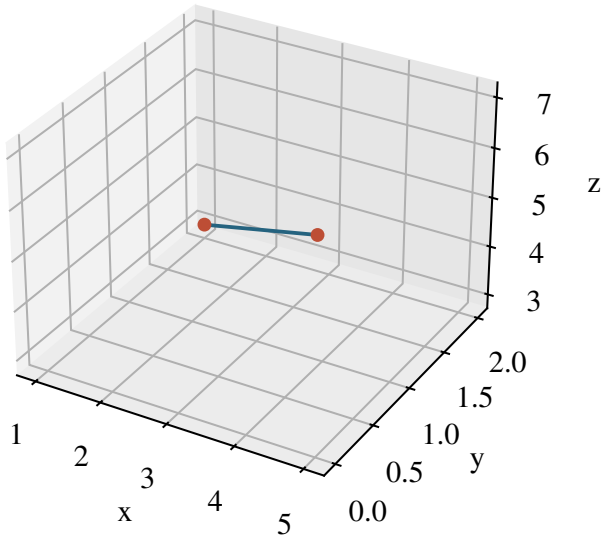
$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}, \qquad \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

■ Worked example

For a=(1,2,3),b=(2,-1,2), λ=2 gives r=(5,0,7). The magnitude of b is $\sqrt{}$(4+1+4)=3. With c=(1,0,-1), b · c=2-2=0, so b and c are perpendicular.

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$$\mathbf{r} = (1, 2, 3) + \lambda(2, -1, 2)$$



Where it goes wrong

- Satisfying two component equations does not guarantee the third.
- Nonparallel lines may be skew.
- A line's direction vector is not its position vector, and a scalar parameter has no coordinate units.

> **Watch out**
>
> **Not true:** Any two nonparallel lines in three dimensions must intersect. In three dimensions nonparallel lines can be skew: they do not meet.

Applying it

- State whether a requested angle is acute, directed or an angle between lines.
- Use the syllabus-specific plane or distance method only when it is actually required for that unit.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Direction vector**: A nonzero vector giving the direction of a line.
- A line has r=a+λb with position vector a and nonzero direction b.
- Watch for: satisfying two component equations does not guarantee the third.

FP3 · Matrix transformations, inverses and eigenvalues

Where do the basis vectors go?

- A map stretches one direction and shears another. A matrix records the transformation of each basis vector.
- This lesson studies **determinant** 行列式: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.

The method

- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Transform column vectors by left multiplication.
- The rightmost matrix acts first in a product.

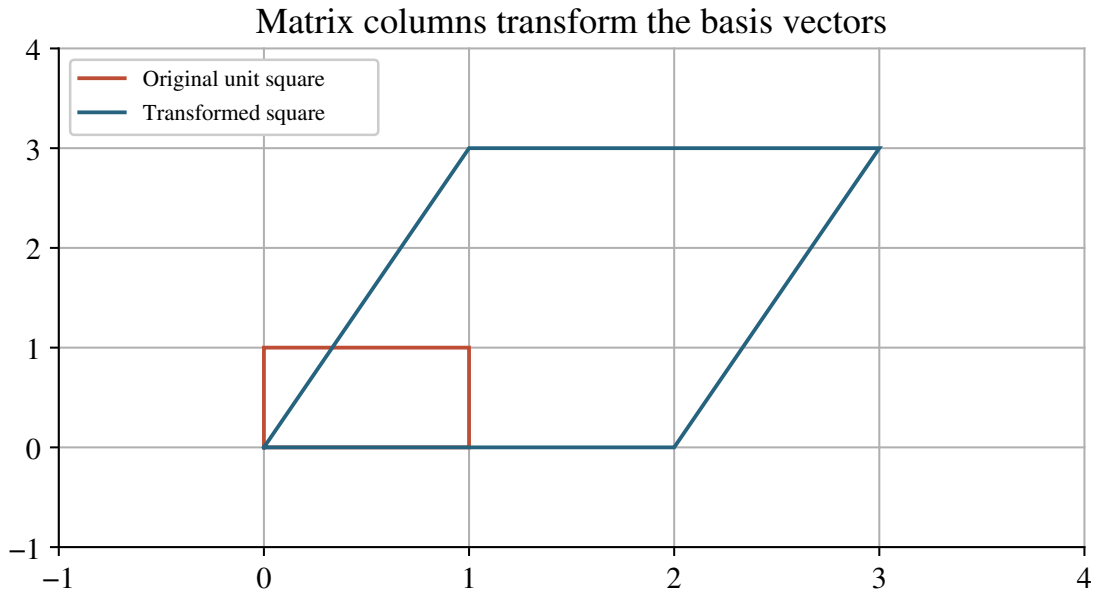
A worked case

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

■ **Worked example**

For A=[[2,1],[0,3]], det A=6. A(1,2)=(4,6). Its inverse is [[1/2,-1/6],[0,1/3]]. The characteristic equation is (2-λ)(3-λ)=0, so the eigenvalues are 2 and 3.

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Where it goes wrong

- Matrix multiplication is usually not commutative.
- A determinant of zero means the transformation loses a dimension and has no inverse.
- Eigenvalues and diagonalisation are advanced scope, not ordinary GCSE vector content.

■ Watch out

Not true: Every square matrix has an inverse. A square matrix with determinant 0 is singular and has no inverse.

Applying it

- Find an eigenvector by solving $(A-\lambda I)v=0$ with $v\neq 0$.
- Explain the geometrical meaning: this vector keeps its line direction under the transformation.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit FP3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Determinant**: A scalar that determines whether a square matrix is invertible and, in two dimensions, its signed area factor.
- A 2×2 matrix [[a,b],[c,d]] has determinant ad-bc and inverse [[d,-b],[-c,a]]/(ad-bc) if the determinant is nonzero.
- Watch for: matrix multiplication is usually not commutative.

M1 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
- This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.

The method

- Choose a positive direction and draw a force diagram.
- Use resultant F=ma.
- For constant acceleration, v=u+at and s=ut+at^2/2.
- A particle model ignores size; a smooth surface ignores friction.

A worked case

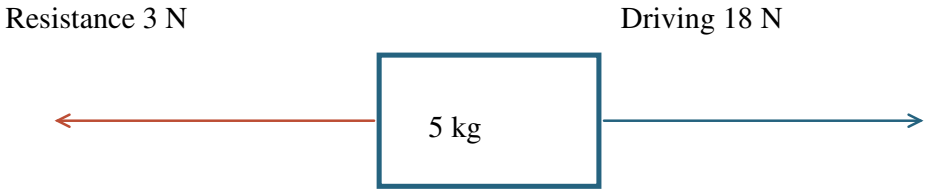
$$F_{\text{resultant}} = ma, \qquad s = ut + \frac{1}{2}at^2$$

■ Worked example

A 5 kg trolley has driving force 18 N and resistance 3 N. The resultant is 15 N, so a=15/5=3 m/s^2. Starting from rest for 4 s gives v=0+3×4=12 m/s and s=0×4+3×4^2/2=24 m.

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Identify the resultant before using F = ma



Where it goes wrong

- The normal reaction need not equal weight on a slope.
- Connected-particle models need one equation per particle and a consistent acceleration relation.
- The constant-acceleration formulae cannot be used for arbitrary variable acceleration.

■ Watch out

Not true: The constant-acceleration formulae apply unchanged when acceleration varies with time. The suvat formulae need constant acceleration; when acceleration varies, use calculus.

Applying it

- Explain what each idealisation permits and what it leaves out.
- For a velocity-time graph, gradient represents acceleration and signed area represents displacement.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Resultant force**: The vector sum of all external forces acting on the modelled object.
- Choose a positive direction and draw a force diagram.
- Watch for: the normal reaction need not equal weight on a slope.

M1 · Vectors and transformation geometry

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** 合向量: The vector sum representing the combined displacement or force.

The method

- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

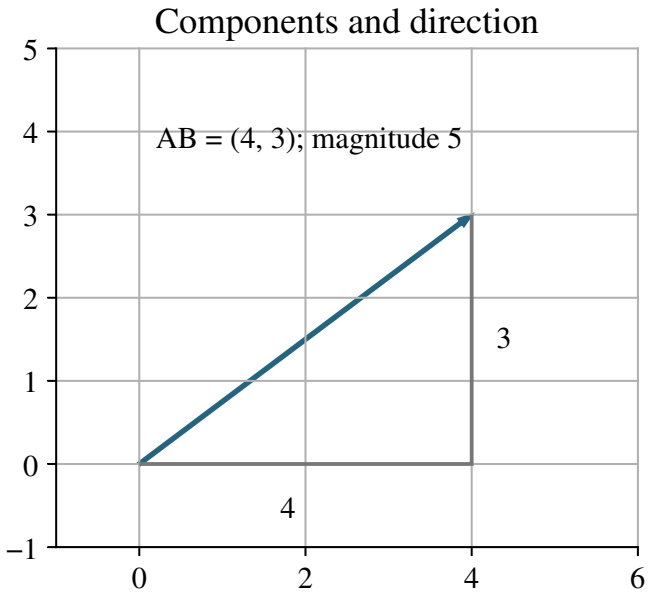
A worked case

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \qquad |\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$$

■ Worked example

With a=(4,1) and b=(1,3), a+b=(5,4). From A=(1,2) to B=(5,5), displacement AB=(4,3). Its magnitude is $\sqrt{(4^2+3^2)}=5$.

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Where it goes wrong

- The order of subtraction matters: BA=-AB.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

> ■ **Watch out**
>
> **Not true:** AB and BA have the same components. BA goes the opposite way to AB, so BA=−AB: every component changes sign.

Applying it

- Use vector components, scalar multiples and magnitudes for displacements and forces.
- This unit does not require the scalar product.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Resultant**: The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: BA=-AB.

M1 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

The method

- For equilibrium, both the resultant force and resultant moment are zero.
- Take moments about a useful pivot to remove unknown forces through that point.
- A combined centre of mass is a mass-weighted mean position.

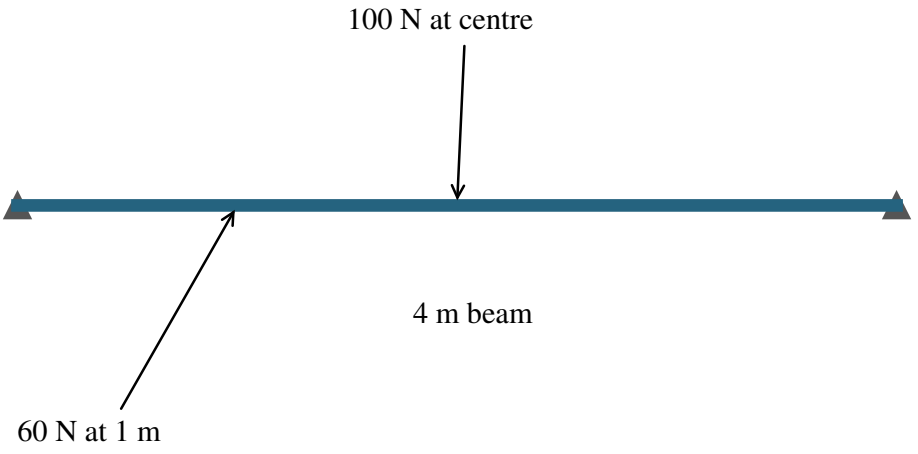
A worked case

$$M = Fd_{\perp}, \qquad \sum M = 0, \qquad \sum \mathbf{F} = \mathbf{0}$$

■ **Worked example**

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give 4R_right=100×2+60×1=260, so R_right=65 N. Vertical equilibrium gives R_left=95 N.

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Where it goes wrong

- Use the perpendicular lever arm, not a diagonal distance to the point of application.
- A zero resultant force alone does not prevent rotation.
- A support that would need a negative normal reaction may lose contact.

> ■ **Watch out**
>
> **Not true:** Zero resultant force by itself is enough for a rigid body's equilibrium. Equilibrium also needs the resultant moment to be zero; two equal opposite forces on different lines still turn the body.

Applying it

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution.
- A removed region has negative mass in that calculation, not a new physical negative material.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Moment**: Force multiplied by the perpendicular distance from its line of action to a pivot.
- For equilibrium, both the resultant force and resultant moment are zero.
- Watch for: use the perpendicular lever arm, not a diagonal distance to the point of application.

M1 · Inclined planes, friction and connected particles

Is friction already at its maximum?

- A crate on a slope is at rest. Friction can adjust to balance the force down the slope; it need not already be at its maximum.
- This lesson studies **limiting friction** 极限摩擦力: The maximum static friction before slipping, equal to μ times the normal reaction in the model.

The method

- Resolve parallel and perpendicular to the plane.
- Weight components are mg $\sin\theta$ and mg $\cos\theta$.
- Static friction satisfies F$\leq\mu$R and equals μR only at the limiting case.
- Write separate equations for connected particles.

A worked case

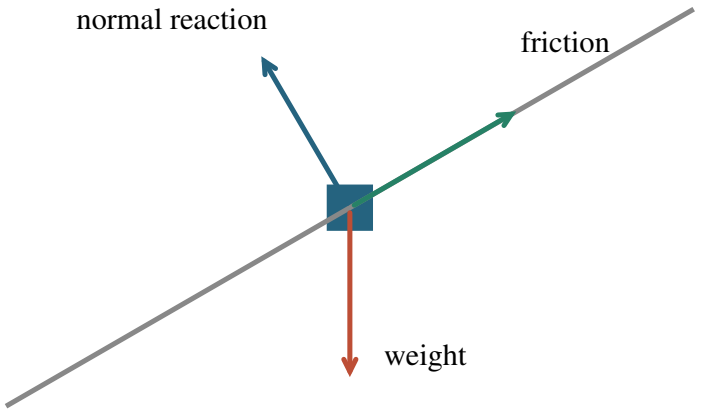
$$R = mg\cos\theta, \qquad F \leq \mu R$$

■ Worked example

A 5 kg crate on a 30° slope has R=5×9.8×cos30°≈42.44 N. The downhill weight component is 24.5 N. With μ=0.6, maximum friction≈25.46 N, so equilibrium is possible with friction 24.5 N.

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Resolve weight along and normal to the slope



Where it goes wrong

- Friction opposes motion or the tendency to move, not always the coordinate direction.
- A taut light inextensible string over a smooth pulley gives equal tension and a common acceleration magnitude; each assumption has a job.

> ■ **Watch out**
>
> **Not true:** Static friction is exactly μR even when the body is not about to slip. Static friction is at most μR; it equals μR only when slipping is about to happen.

Applying it

- Start with force diagrams and a proposed direction of motion.
- If the calculated direction conflicts with the friction assumption, revisit the model instead of keeping inconsistent signs.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit M1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Limiting friction**: The maximum static friction before slipping, equal to μ times the normal reaction in the model.
- Resolve parallel and perpendicular to the plane.
- Watch for: friction opposes motion or the tendency to move, not always the coordinate direction.

M2 · Kinematics and Newton's laws

Which force accelerates the trolley?

- A trolley accelerates while friction opposes its motion. The engine's force alone does not equal mass times acceleration.
- This lesson studies **resultant force** 合力: The vector sum of all external forces acting on the modelled object.

The method

- Choose a positive direction and draw a force diagram.
- Use resultant F=ma.
- For constant acceleration, v=u+at and s=ut+at^2/2.
- A particle model ignores size; a smooth surface ignores friction.

A worked case

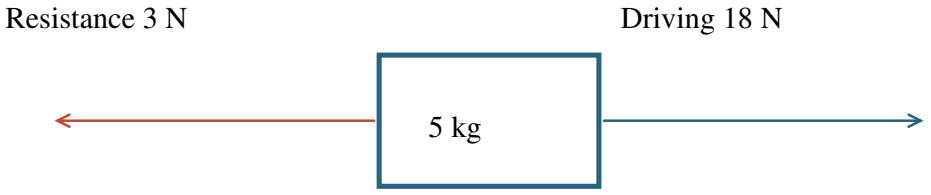
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■ **Worked example**

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Identify the resultant before using F = ma



Where it goes wrong

- The normal reaction need not equal weight on a slope.
- Connected-particle models need one equation per particle and a consistent acceleration relation.
- The constant-acceleration formulae cannot be used for arbitrary variable acceleration.

■ Watch out

Not true: The constant-acceleration formulae apply unchanged when acceleration varies with time. The suvat formulae need constant acceleration; when acceleration varies, use calculus.

Applying it

- Explain what each idealisation permits and what it leaves out.
- For a velocity-time graph, gradient represents acceleration and signed area represents displacement.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Resultant force**: The vector sum of all external forces acting on the modelled object.
- Choose a positive direction and draw a force diagram.
- Watch for: the normal reaction need not equal weight on a slope.

M2 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

The method

- For equilibrium, both the resultant force and resultant moment are zero.
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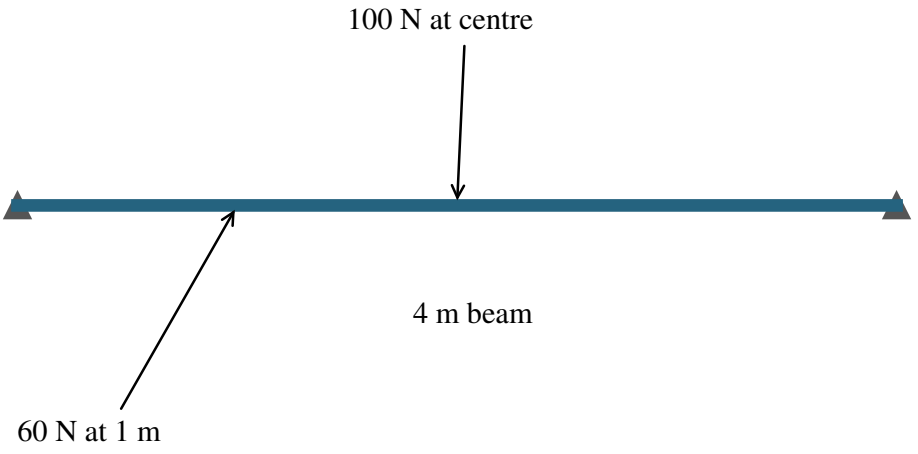
A worked case

$$M = Fd_{\perp}, \qquad \sum M = 0, \qquad \sum \mathbf{F} = \mathbf{0}$$

■ Worked example

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Where it goes wrong

- Use the perpendicular lever arm, not a diagonal distance to the point of application.
- A zero resultant force alone does not prevent rotation.
- A support that would need a negative normal reaction may lose contact.

> ■ **Watch out**
>
> **Not true:** Zero resultant force by itself is enough for a rigid body's equilibrium. Equilibrium also needs the resultant moment to be zero; two equal opposite forces on different lines still turn the body.

Applying it

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution.
- A removed region has negative mass in that calculation, not a new physical negative material.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Moment**: Force multiplied by the perpendicular distance from its line of action to a pivot.
- For equilibrium, both the resultant force and resultant moment are zero.
- Watch for: use the perpendicular lever arm, not a diagonal distance to the point of application.

M2 · Work, energy, power and collisions

What does a slower rebound tell us?

- A ball rebounds slower after hitting a wall. Momentum and kinetic energy answer different parts of the collision story.
- This lesson studies **coefficient of restitution** 恢复系数: The ratio of relative separation speed to relative approach speed along the line of impact.

The method

- Work is force times displacement in its direction.
- Kinetic energy is $mv^2/2$ and gravitational potential energy is mgh.
- Momentum is conserved for an isolated collision; restitution gives the relative speed relation, not conservation of kinetic energy.

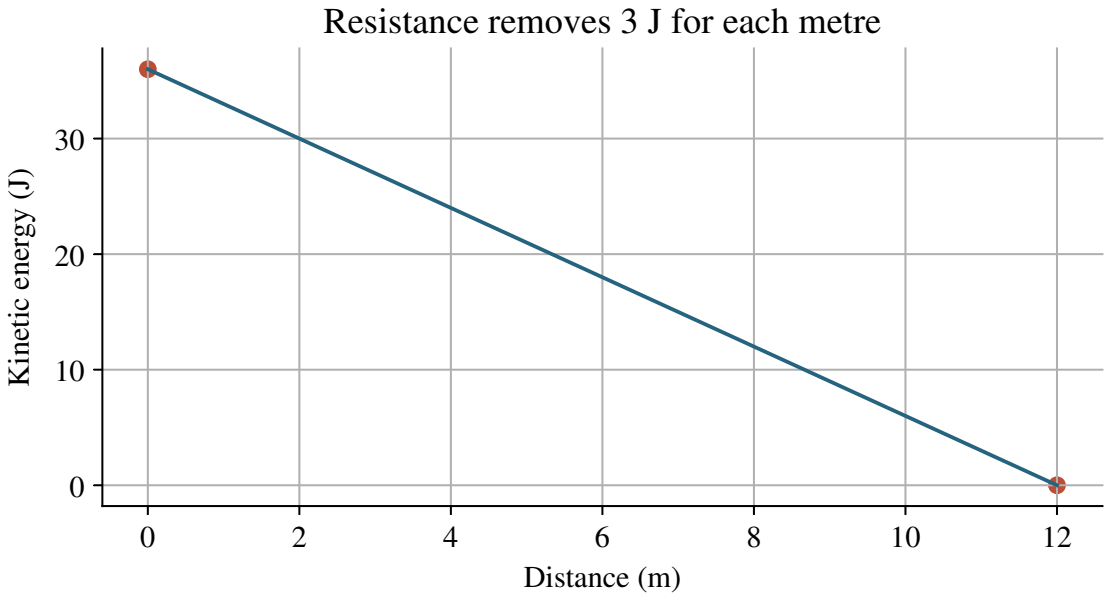
A worked case

$$E_k = \frac{1}{2}mv^2, \qquad W = Fs, \qquad P = \frac{dE}{dt}$$

■ Worked example

A 2 kg particle moving at 6 m/s has kinetic energy $2\times6^2/2=36$ J. If a constant opposing force of 3 N stops it, work=3s=36, giving s=12 m. A wall bounce with approach speed 5 and e=0.6 has rebound speed 3 m/s in the opposite direction.

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Where it goes wrong

- Conserve momentum only when the external impulse is negligible.
- Kinetic energy is generally lost in an inelastic collision.
- Power is a rate of energy transfer, not an amount of energy.

Watch out

Not true: Every collision conserves kinetic energy. Kinetic energy is conserved only in perfectly elastic collisions; momentum is conserved in every collision.

Applying it

- State the direction signs before writing collision equations.
- For a connected motion problem, choose the whole system for energy when internal tensions cancel; account for frictional work explicitly.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Coefficient of restitution**: The ratio of relative separation speed to relative approach speed along the line of impact.
- Work is force times displacement in its direction.
- Watch for: conserve momentum only when the external impulse is negligible.

M2 · Projectile motion and variable acceleration

Why does horizontal motion share vertical time?

- A ball is launched at an angle. Its horizontal and vertical motion share time but follow different equations.
- This lesson studies **projectile** 抛体: A particle moving under gravity after launch in a model that neglects air resistance.

The method

- Resolve initial velocity into horizontal u cosθ and vertical u sinθ.
- With no air resistance, horizontal acceleration is zero and vertical acceleration is -g.
- Use the same time in both components.

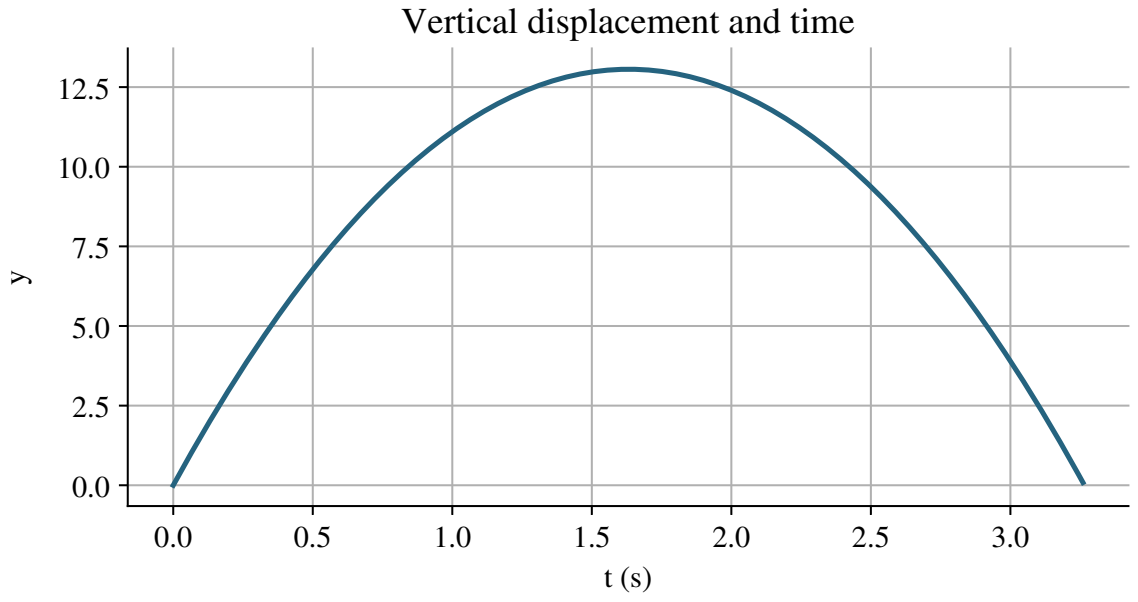
A worked case

$$x = u_x t, \qquad y = u_y t - \frac{1}{2} g t^2$$

■ **Worked example**

For components u_x=12,u_y=16 and g=9.8, at t=2 s the horizontal displacement is x=12×2=24 m and vertical displacement is y=16×2-9.8×2^2/2=12.4 m. At the highest point, v_y=0 but v_x remains 12.

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Where it goes wrong

- Zero vertical velocity at the highest point does not mean zero total speed.
- Launch and landing heights need not be equal.
- Do not use a range formula that assumes equal heights without checking them.

> ■ **Watch out**
>
> **Not true:** The total speed of a projectile is zero at its highest point. At the top the vertical velocity is zero, but the horizontal velocity remains, so the speed is not zero.

Applying it

- For variable acceleration, integrate a(t) to get v(t) and use the initial velocity to find the constant, then integrate for displacement.
- Confirm units and the physical time interval.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit M2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Projectile**: A particle moving under gravity after launch in a model that neglects air resistance.
- Resolve initial velocity into horizontal u cosθ and vertical u sinθ.
- Watch for: zero vertical velocity at the highest point does not mean zero total speed.

M3 · Elasticity, circular motion and variable dynamics

Which length stretches the spring?

- A stretched spring stores energy. A particle moving in a circle needs a resultant force towards the centre, even at constant speed.
- This lesson studies **extension** 伸长量: The increase in an elastic string's or spring's length from its natural length.

The method

- For an elastic string, tension=λx/l when stretched, with natural length l and modulus λ.
- Elastic energy=λx^2/(2l).
- For circular motion, radial acceleration=v^2/r; resolve actual forces in the radial direction.

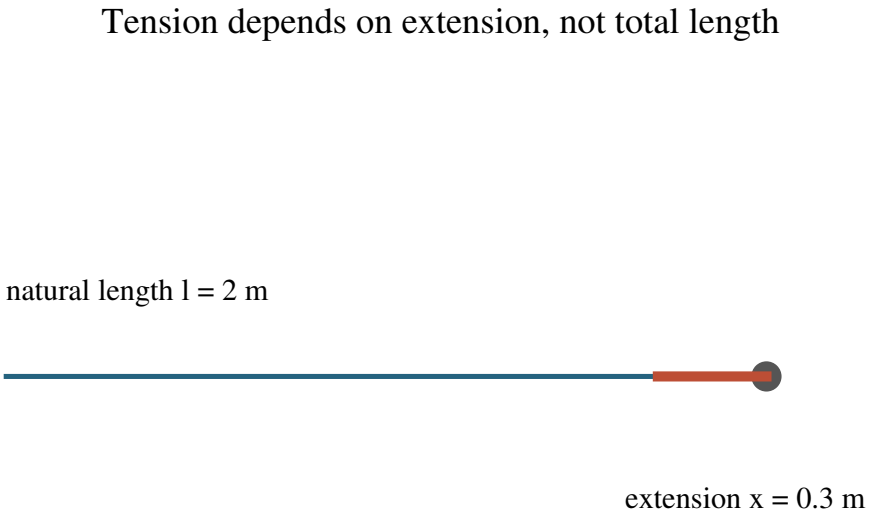
A worked case

$$T = \frac{\lambda x}{l}, \qquad E = \frac{\lambda x^2}{2l}, \qquad F_r = \frac{mv^2}{r}$$

■ **Worked example**

With λ=40 N,l=2 m,x=0.3 m, tension=40×0.3/2=6 N and elastic energy=40×0.3^2/(2×2)=0.9 J. A 2 kg particle at speed 3 m/s on radius 1.5 m needs resultant inward force=2×3^2/1.5=12 N.

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Where it goes wrong

- Centripetal force is the name of the inward resultant, not an extra force to add to the diagram.
- A slack string has no tension.
- Distinguish extension from total stretched length.

> ■ **Watch out**
>
> **Not true:** Centripetal force is an additional force that must be added to all the real forces. The centripetal force is the resultant of the real forces, such as tension or friction, not an extra force.

Applying it

- For variable acceleration, use a=dv/dt or a=v dv/dx when appropriate.
- Combine the chosen motion relation with the force or energy model, then check the permitted range of motion.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Extension**: The increase in an elastic string's or spring's length from its natural length.
- For an elastic string, tension=λx/l when stretched, with natural length l and modulus λ.
- Watch for: centripetal force is the name of the inward resultant, not an extra force to add to the diagram.

M3 · Differential equations and numerical solutions

What does the starting value decide?

- A growing population's rate is proportional to its current size. The rate equation describes a whole family until an initial population is supplied.
- This lesson studies **initial condition** 初始条件: A specified value that selects a particular solution from a family.

The method

- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Use an initial condition to find A.
- Euler's method takes y next=y+h f(x,y), with a chosen step h.

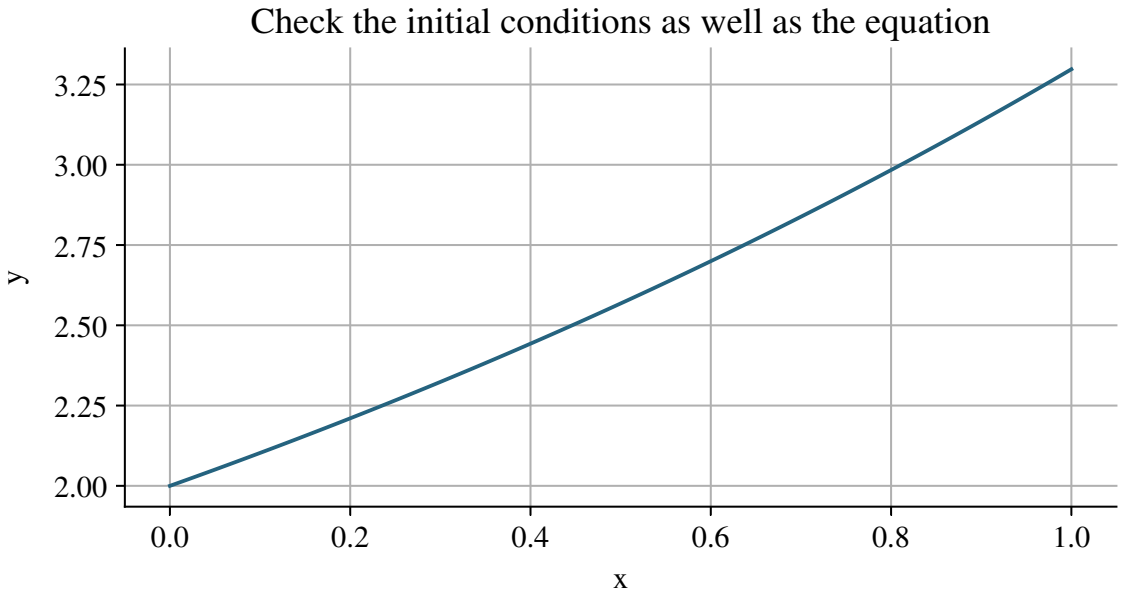
A worked case

$$\frac{dy}{dx} = ky, \qquad y = Ae^{kx}$$

Worked example

With dy/dx=0.5y and y(0)=2, y=2e^(0.5x). Euler with h=0.2 gives y(0.2)≈2+0.2×1=2.2 and y(0.4)≈2.2+0.2×1.1=2.42. The exact second value is about 2.4428.

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Where it goes wrong

- An initial condition determines the integration constant; it is not a replacement for integrating.
- Dividing by y can omit an equilibrium solution y=0.
- Euler accuracy depends on step size and the equation.

> ■ **Watch out**
>
> **Not true:** Every step size gives exactly the same Euler approximation. Euler's method depends on the step size; smaller steps usually give a more accurate value.

Applying it

- For second-order linear equations, combine the complementary function with an appropriate particular integral, then apply the required initial conditions.
- This is Further Pure scope; check the named unit.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit M3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Initial condition**: A specified value that selects a particular solution from a family.
- For dy/dx=ky, separate variables: dy/y=k dx, giving y=Ae^(kx).
- Watch for: an initial condition determines the integration constant; it is not a replacement for integrating.

M3 · Moments, equilibrium and centres of mass

Why does a longer spanner help?

- A long spanner turns a nut more easily. The turning effect depends on perpendicular distance as well as force.
- This lesson studies **moment** 力矩: Force multiplied by the perpendicular distance from its line of action to a pivot.

The method

- For equilibrium, both the resultant force and resultant moment are zero.
- Take moments about a useful pivot to remove unknown forces through that point.
- A combined centre of mass is a mass-weighted mean position.

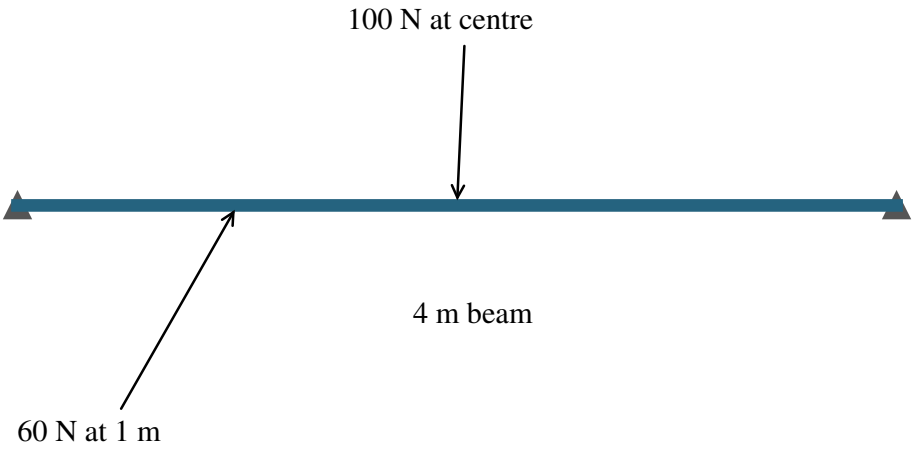
A worked case

$$M = Fd_{\perp}, \qquad \sum M = 0, \qquad \sum \mathbf{F} = \mathbf{0}$$

■ **Worked example**

A uniform 4 m beam weighs 100 N and is supported at both ends. An extra 60 N acts 1 m from the left. Moments about the left give 4R_right=100×2+60×1=260, so R_right=65 N. Vertical equilibrium gives R_left=95 N.

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Where it goes wrong

- Use the perpendicular lever arm, not a diagonal distance to the point of application.
- A zero resultant force alone does not prevent rotation.
- A support that would need a negative normal reaction may lose contact.

> ■ **Watch out**
>
> **Not true:** Zero resultant force by itself is enough for a rigid body's equilibrium. Equilibrium also needs the resultant moment to be zero; two equal opposite forces on different lines still turn the body.

Applying it

- For a lamina made from parts, choose a common origin and tabulate each signed mass-area contribution.
- A removed region has negative mass in that calculation, not a new physical negative material.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit M3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Moment**: Force multiplied by the perpendicular distance from its line of action to a pivot.
- For equilibrium, both the resultant force and resultant moment are zero.
- Watch for: use the perpendicular lever arm, not a diagonal distance to the point of application.

S1 · Data summaries, histograms and interpretation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** 频率密度: Frequency divided by class width, used as histogram height.

The method

- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

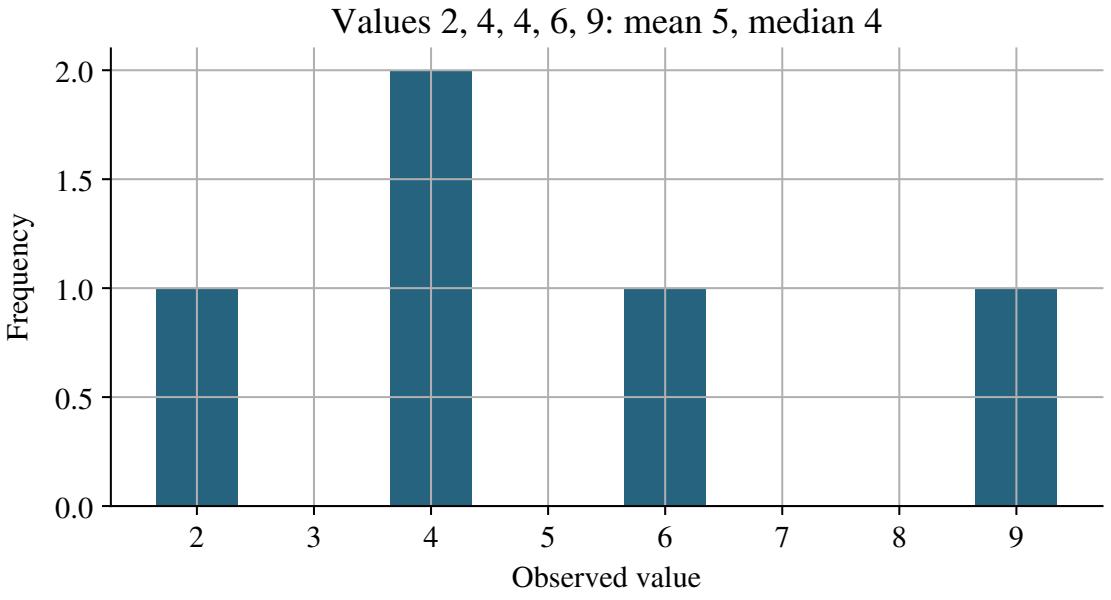
A worked case

$$\overline{x} = \frac{\sum x_i}{n}, \qquad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ Worked example

A class from 10 to 20 with frequency 30 has density 30/10=3. A class from 20 to 40 with frequency 20 has density 20/20=1. Its wider bar must not be mistaken for a larger density. For values 2,4,4,6,9, the median is 4 and mean is 5.

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Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Frequency density**: Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

S1 · Probability, trees and conditional reasoning

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability** 条件概率: The probability of an event after restricting the sample space to a stated condition.

The method

- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is P(A given B)=P(A∩B)/P(B), for P(B)>0.

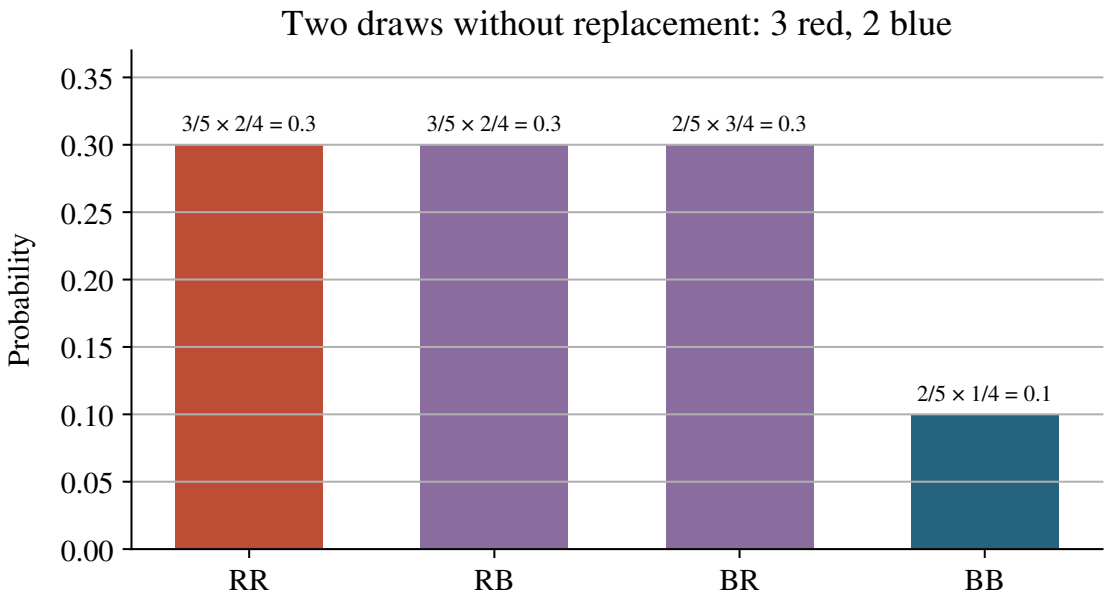
A worked case

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

■ Worked example

Without replacement, P(two red)=3/5×2/4=3/10. P(one of each)=3/5×2/4+2/5×3/4=3/5. If P(A∩B)=0.12 and P(B)=0.3, P(A given B)=0.4.

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Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

> ■ **Watch out**
>
> **Not true:** Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B)=0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Conditional probability**: The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

S1 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.

The method

- A linear model y=a+bx has intercept a and slope b with contextual units.
- Inspect residuals and the data range.
- For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.

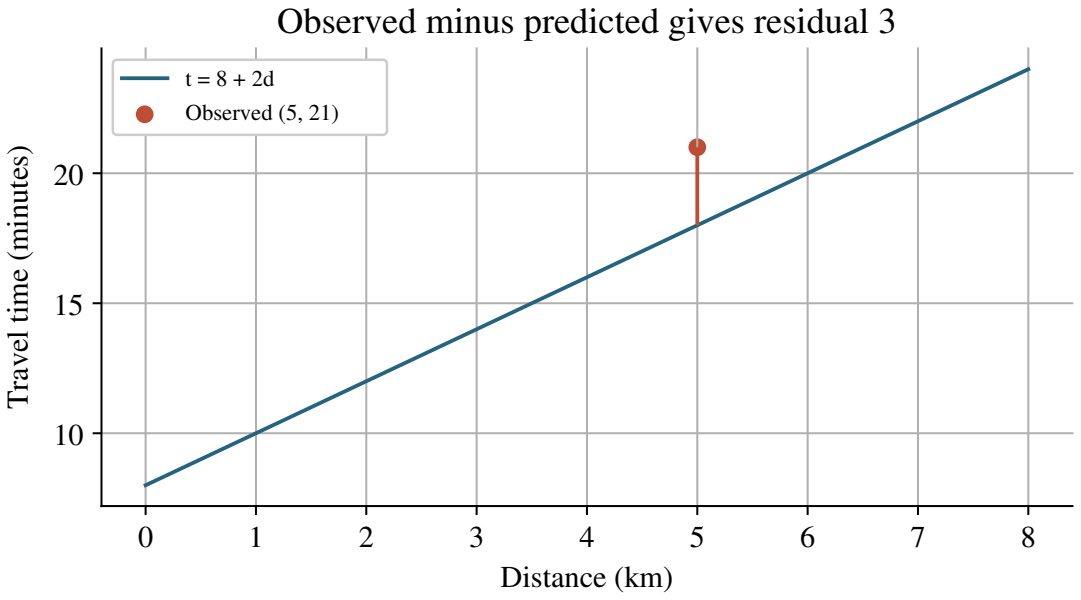
A worked case

$$\widehat{t} = 8 + 2d, \qquad e = t - \widehat{t}$$

■ **Worked example**

The fitted model t=8+2d predicts 18 minutes at d=5 km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes 1000×1.05^3=1157.625 after 3 years.

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Where it goes wrong

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

■ Watch out

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

Applying it

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S1.

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■ **Key idea**

- **Residual**: The observed value minus the value predicted by a fitted model.
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Watch for: A good fit does not prove a causal mechanism.

S1 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

- For a discrete random variable, sum the probabilities to 1 and calculate E(X)=sum xP(X=x).
- For the normal model use z=(x-μ)/σ and calculate an appropriate area.

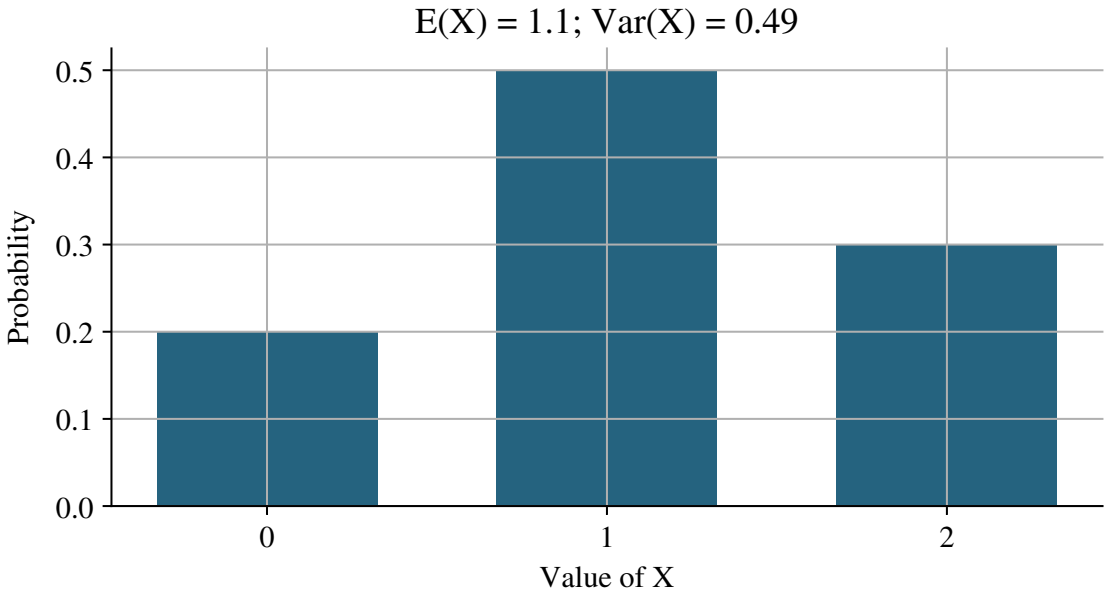
A worked case

$$E(X) = \sum xP(X = x), \quad z = \frac{x - \mu}{\sigma}$$

■ **Worked example**

If X takes 0,1,2 with probabilities 0.2,0.5,0.3, E(X)=0×0.2+1×0.5+2×0.3=1.1. Also $E(X^2)$=0+0.5+1.2=1.7, so Var(X)=1.7-1.1^2=0.49. For μ=100,σ=15, a value of 130 has z=2.

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Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

Watch out

Not true: A random variable with three possible values must be binomial. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- S1 includes discrete random variables and the normal distribution.
- Named binomial and Poisson models are S2 content.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Expected value**: The probability-weighted mean of a random variable.
- For a discrete random variable, sum the probabilities to 1 and calculate E(X)=sum xP(X=x).
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

S2 · Binomial, normal and Poisson models

What makes a count predictable?

- A quality inspector counts defective items. The number is random, but a model can describe its likely range.
- This lesson studies **expected value** 期望值: The probability-weighted mean of a random variable.

The method

- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- E(X)=np and Var(X)=np(1-p).
- For a normal model use z=(x-μ)/σ.
- A Poisson model describes counts with a constant rate and appropriate independence assumptions.

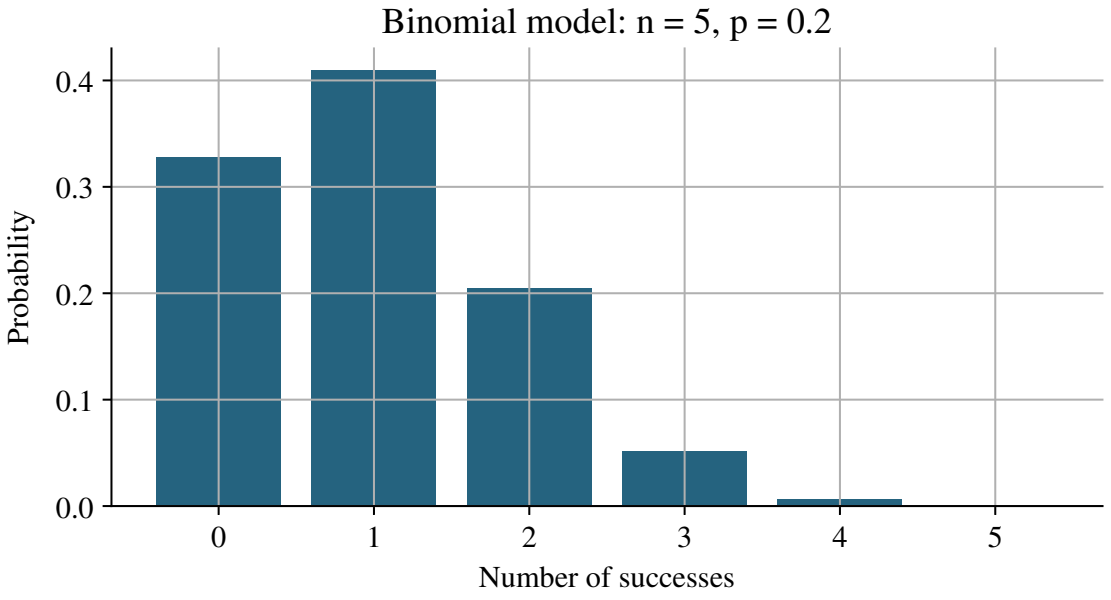
A worked case

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad X \sim B(n, p)$$

■ **Worked example**

For X binomial(5,0.2), P(X=0)=0.8^5=0.32768, E(X)=1 and Var(X)=0.8. For a normal quantity with μ=100,σ=15, the value 130 has z=2. A Poisson mean of 3 per hour gives mean 6 over two hours.

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Where it goes wrong

- Not every count is binomial: changing p or dependence can invalidate it.
- For a continuous variable, the probability of one exact value is zero.
- Continuity correction matters when approximating a discrete distribution by a normal one.

> ■ **Watch out**
>
> **Not true:** Every count of successes has a binomial distribution regardless of dependence. The binomial model needs independent trials with a fixed probability; dependent trials break it.

Applying it

- Write the event as an inequality before using calculator distribution functions.
- Distinguish $P(X<k)$, $P(X\leq k)$ and a tail complement.
- State assumptions in context.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit S2.

■ Key idea

- **Expected value**: The probability-weighted mean of a random variable.
- A binomial model needs fixed n, independent trials, two outcomes and constant p.
- Watch for: not every count is binomial: changing p or dependence can invalidate it.

S2 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
- This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.

The method

- Identify the population, sampling unit and frame.
- Distinguish random, systematic, stratified, quota and opportunity sampling.
- Missing data is not zero; verify units, dates and variable definitions before comparing samples.

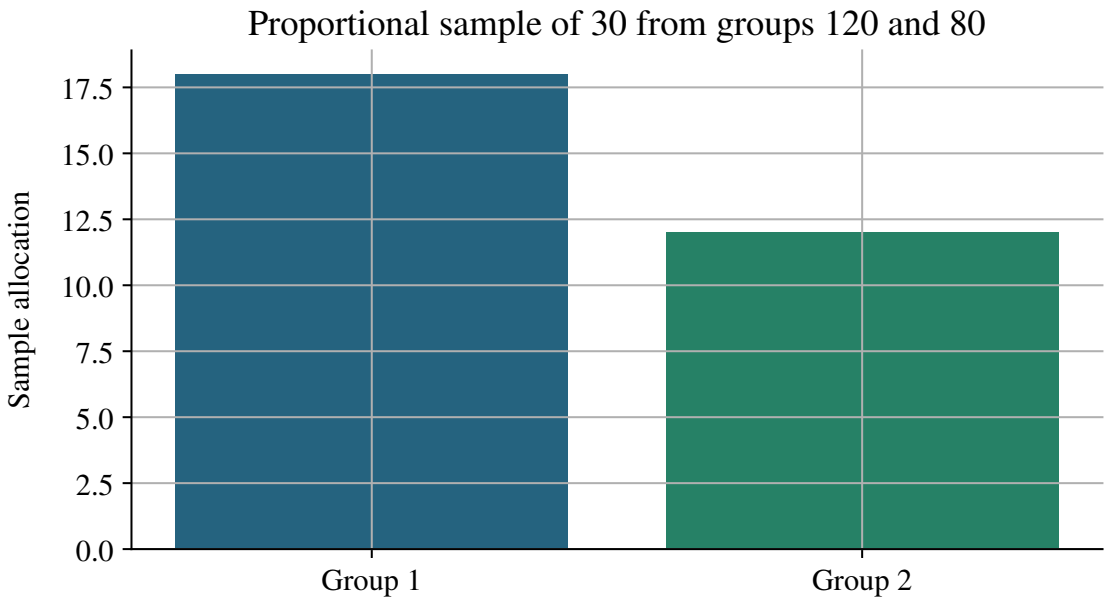
A worked case

$$n_i = n\frac{N_i}{N}$$

■ Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30\times120/200=18$ from the first and 12 from the second. Random selection is then needed within each group.

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Where it goes wrong

- A large biased sample remains biased.
- Stratification is not the same as selecting whoever is available from each group.
- AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.

Watch out

Not true: Doubling a biased sample removes its selection bias. A bigger sample collected the same biased way keeps the same bias; only better sampling removes it.

Applying it

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation.
- Save the decisions with the analysis.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Sampling frame**: The list or population definition from which a sample is selected.
- Identify the population, sampling unit and frame.
- Watch for: A large biased sample remains biased.

S2 · Hypothesis testing and contextual conclusions

Could chance explain the result?

- A factory claims that only 10% of items are defective. A sample contains more defects, but chance alone may explain some difference.
- This lesson studies **significance level** 显著性水平: The chosen probability threshold for rejecting a null hypothesis.

The method

- State H_0 and H_1 in population parameters before inspecting the outcome.
- Calculate the appropriate tail probability under H_0.
- Reject H_0 when the evidence meets the specified significance rule; otherwise say there is insufficient evidence.

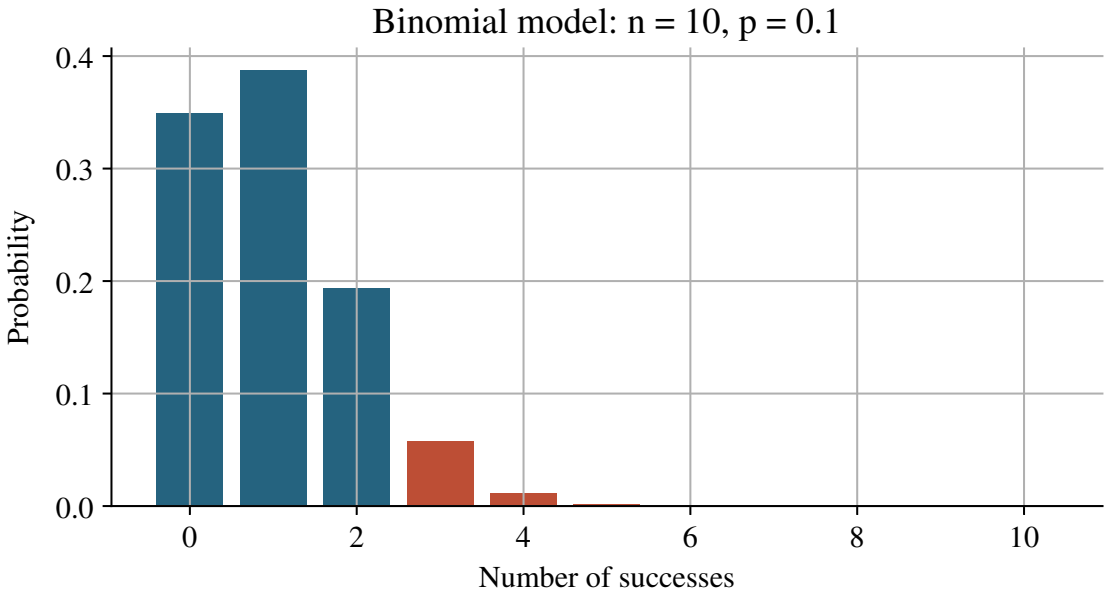
A worked case

$$H_0 : p = p_0, \qquad H_1 : p > p_0$$

■ Worked example

For H_0:p=0.1 and H_1:p>0.1 with n=10, observing at least 3 defects has probability 1-(0.9^10+10×0.1×0.9^9+45×0.1^2×0.9^8)≈0.070191. At 5%, this is insufficient evidence that the defect rate exceeds 10%.

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Where it goes wrong

- Failing to reject H_0 is not proof that H_0 is true.
- Choose the tail from H_1, not from whichever tail gives a small result.
- Statistical significance does not measure the practical size of an effect.

> ■ **Watch out**
>
> **Not true:** Failing to reject a null hypothesis proves it is true. Not rejecting the null hypothesis only means there is not enough evidence against it.

Applying it

- Finish with a sentence about the population and the original claim.
- State the model's assumptions and consider whether the sampling procedure supports them.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Significance level**: The chosen probability threshold for rejecting a null hypothesis.
- State H_0 and H_1 in population parameters before inspecting the outcome.
- Watch for: failing to reject H_0 is not proof that H_0 is true.

S2 · Continuous distributions and density functions

Is a height the same as a probability?

- A waiting time can take any value in an interval. Its probability comes from area, not the graph's height at one instant.
- This lesson studies **probability density** 概率密度: A nonnegative function whose integral over an interval gives its probability.

The method

- A density f must be nonnegative and integrate to 1 over its support.
- The cumulative distribution F(x) is the integral up to x.
- For a continuous variable P(X=x)=0 and E(X) is the integral of xf(x).

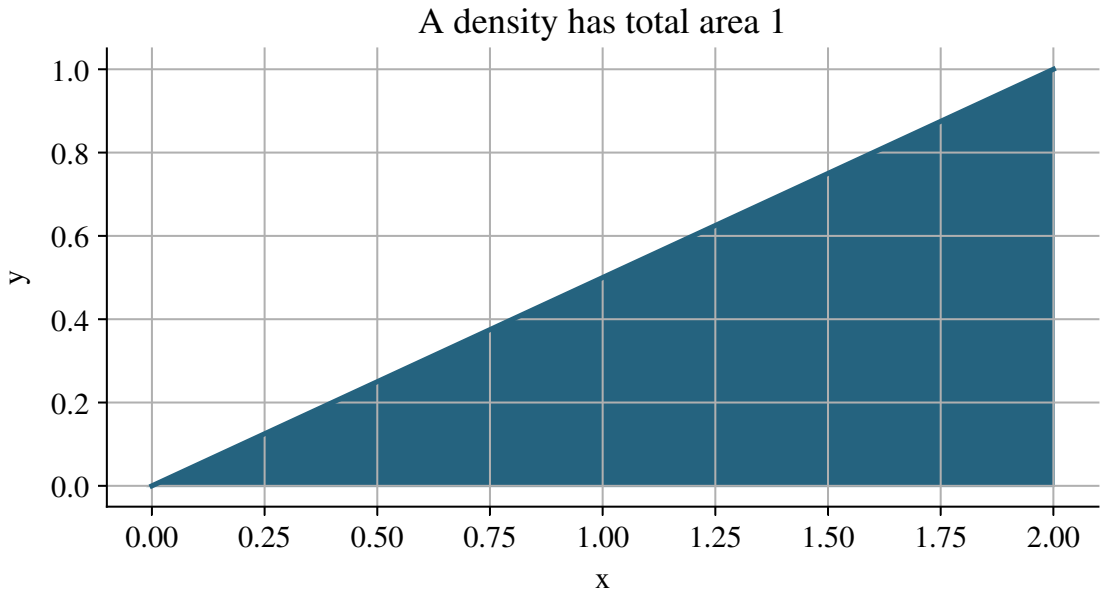
A worked case

$$\int f(x)\,dx = 1, \qquad E(X) = \int x f(x)\,dx$$

■ **Worked example**

Let f(x)=kx on 0≤x≤2 and zero elsewhere. Normalisation gives integral_0^2 kx dx=2k=1, so k=1/2. Then P(X≤1)=integral_0^1 x/2 dx=1/4 and E(X)=integral_0^2 $x^2/2$ dx=4/3.

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Where it goes wrong

- Density can exceed 1 without being invalid; total area must equal 1.
- A cumulative distribution cannot decrease.
- Include the zero-density region outside the support when defining a complete model.

> ■ **Watch out**
>
> **Not true:** A continuous variable has P(X=x) equal to the density height f(x). For a continuous variable P(X=x)=0; probabilities are areas under the density curve.

Applying it

- For a uniform distribution, density is reciprocal interval length.
- Find medians or percentiles using the cumulative probability, and use a support-aware integral for moments.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S2.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

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■ **Key idea**

- **Probability density**: A nonnegative function whose integral over an interval gives its probability.
- A density f must be nonnegative and integrate to 1 over its support.
- Watch for: density can exceed 1 without being invalid; total area must equal 1.

S3 · Confidence intervals, chi-square and regression

How uncertain is the sample mean?

- Two samples give different means. We need a statement about sampling uncertainty, not a promise that one sample equals the population.
- This lesson studies **confidence interval** 置信区间: An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.

The method

- For a normal mean with known σ, a 95% interval is sample mean$\pm 1.96\sigma/\sqrt{}$n.
- For chi-square, sum (observed-expected)^2/expected with the correct degrees of freedom.
- Choose a test that fits the data and assumptions.

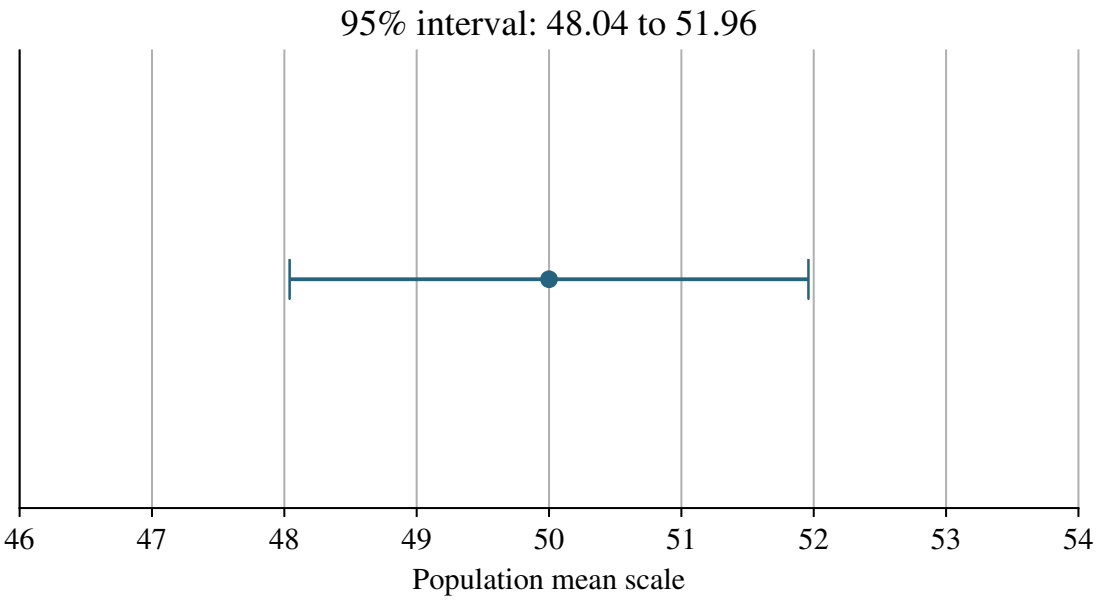
A worked case

$$\overline{x} \pm 1.96\frac{\sigma}{\sqrt{n}}, \qquad \chi^2 = \sum \frac{(O-E)^2}{E}$$

■ **Worked example**

With mean=50,σ=10,n=100, standard error=1 and the 95% interval is [48.04,51.96]. For observed counts 30 and 20 against expected 25 and 25, chi-square=25/25+25/25=2, before deciding degrees of freedom and the rejection threshold.

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Where it goes wrong

- A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.
- Expected counts and estimated parameters affect the validity and degrees of freedom of chi-square tests.

■ Watch out

Not true: A strong correlation alone establishes a causal relationship. Correlation shows association; a third variable or chance can explain it, so causation needs an experiment or other evidence.

Applying it

- Report uncertainty in the context.
- In regression, inspect residuals, avoid unjustified extrapolation, and distinguish a fitted relationship from a causal explanation.
- S3 and IB AI HL require different verified scope boundaries.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit S3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

Key idea

- **Confidence interval**: An interval calculated by a procedure designed to cover the population parameter at a stated long-run rate.
- For a normal mean with known σ, a 95% interval is sample mean$\pm 1.96\sigma/\sqrt{\text{n}}$.
- Watch for: A 95% confidence procedure does not give a 95% probability that a fixed population mean moves within this one interval.

S3 · Sampling and a large data set

Who is missing from the data?

- A weather database has a missing entry and several stations. Treating each row as identical can distort a comparison.
- This lesson studies **sampling frame** 抽样框: The list or population definition from which a sample is selected.

The method

- Identify the population, sampling unit and frame.
- Distinguish random, systematic, stratified, quota and opportunity sampling.
- Missing data is not zero; verify units, dates and variable definitions before comparing samples.

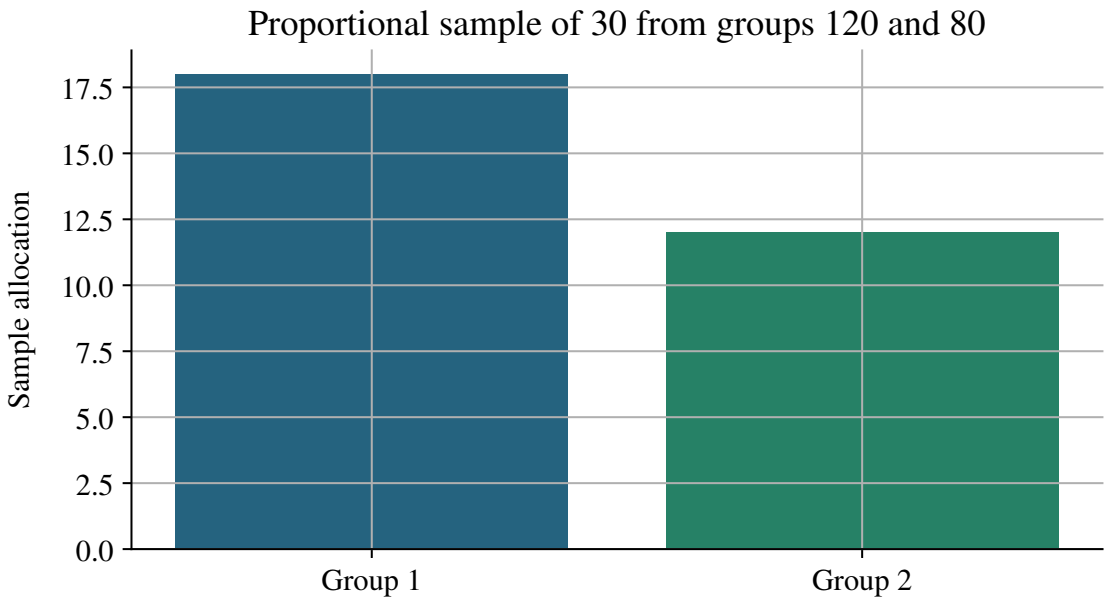
A worked case

$$n_i = n\frac{N_i}{N}$$

Worked example

A population has 120 students in one group and 80 in another. A proportional stratified sample of 30 needs $30\times120/200=18$ from the first and 12 from the second. Random selection is then needed within each group.

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Where it goes wrong

- A large biased sample remains biased.
- Stratification is not the same as selecting whoever is available from each group.
- AQA large-data-set familiarity requires the actual supplied data and metadata, not invented weather values.

■ Watch out

Not true: Doubling a biased sample removes its selection bias. A bigger sample collected the same biased way keeps the same bias; only better sampling removes it.

Applying it

- For AQA, use the official large data set in a supervised spreadsheet task: identify a variable, justify a comparison, inspect missing values, create a display and explain a limitation.
- Save the decisions with the analysis.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit S3.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Sampling frame**: The list or population definition from which a sample is selected.
- Identify the population, sampling unit and frame.
- Watch for: A large biased sample remains biased.

S3 · Regression, financial models and residuals

Is the prediction supported by the data?

- A line predicts bus travel time from distance. A small calculation error matters less than using a model outside its evidence.
- This lesson studies **residual** 残差: The observed value minus the value predicted by a fitted model.

The method

- A linear model y=a+bx has intercept a and slope b with contextual units.
- Inspect residuals and the data range.
- For repeated percentage change use a geometric model; for a loan distinguish principal, rate, repayment and period.

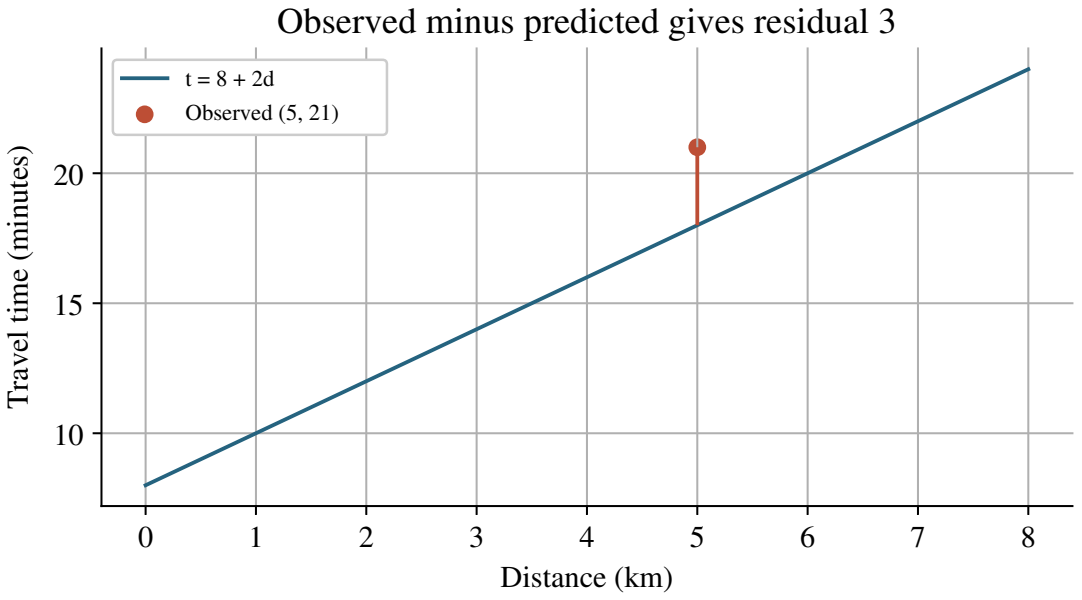
A worked case

$$\widehat{t} = 8 + 2d, \qquad e = t - \widehat{t}$$

■ Worked example

The fitted model t=8+2d predicts 18 minutes at d=5 km. If the observed time is 21, the residual is 3 minutes. A ¥1000 deposit at 5% compound annual interest becomes 1000×1.05^3=1157.625 after 3 years.

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Where it goes wrong

- A good fit does not prove a causal mechanism.
- A correlation coefficient measures linear association, not the gradient.
- Calculator output must be translated into a model, checked and interpreted.

■ Watch out

Not true: A high correlation permits reliable extrapolation to any distance. A model fitted to data is reliable only near that data; extrapolating far outside it is unreliable.

Applying it

- In IB AI, record the data source, domain, assumptions and calculator method.
- Compare an alternative model and judge predictions against residuals.
- A financial answer must state payment timing and whether interest is compounded.

In your course

- Pearson Edexcel International A Level Further Mathematics, unit S3.

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■ **Key idea**

- **Residual**: The observed value minus the value predicted by a fitted model.
- A linear model y=a+bx has intercept a and slope b with contextual units.
- Watch for: A good fit does not prove a causal mechanism.

D1 · Algorithms, networks and linear programming

Cheapest network or shortest route?

- A school must connect buildings with cable. The shortest route between two buildings and the cheapest whole network are different problems.
- This lesson studies **minimum spanning tree** 最小生成树: A connected cycle-free network joining every vertex with the smallest possible total edge weight.

The method

- Kruskal selects edges in increasing weight while avoiding cycles.
- Dijkstra updates shortest tentative distances from a start.
- Linear programming optimizes a linear objective over a feasible region; inspect vertices and integer restrictions when required.

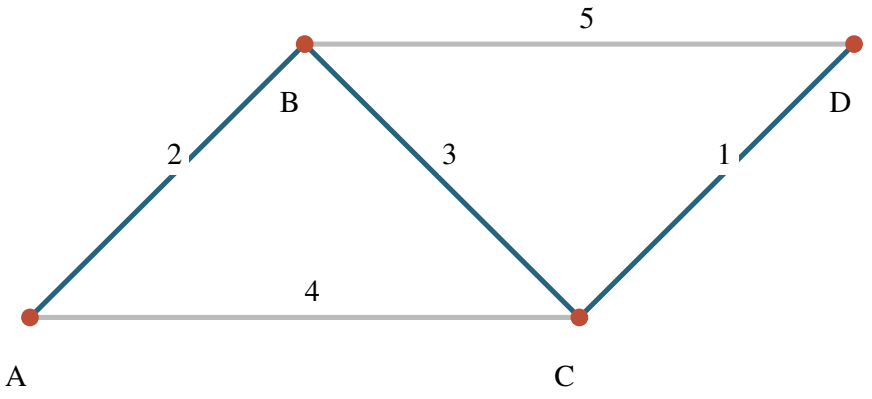
A worked case

$$Z = 3x + 2y, \qquad x + y \leq 4, \quad x \leq 2, \quad x, y \geq 0$$

■ **Worked example**

For vertices A,B,C,D and edges AB=2,BC=3,AC=4,CD=1,BD=5, Kruskal selects CD,AB,BC for total 6. The shortest A-to-D route is A-B-C-D, also 6, but that equality is incidental. Maximize 3x+2y with x+y≤4,x≤2,x,y≥0: the best vertex is (2,2), value 10.

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Where it goes wrong

- A spanning tree has no cycles and joins all vertices.
- The largest single edge is not automatically excluded from every optimal solution.
- A shortest-path algorithm cannot replace a minimum-spanning-tree algorithm.

> ■ **Watch out**
>
> **Not true:** A shortest path between two vertices must also be a minimum spanning tree of the whole network. A shortest path joins two vertices; a minimum spanning tree connects every vertex at least total weight, so they answer different questions.

Applying it

- For critical paths, calculate earliest and latest event times and identify zero-float activities.
- State units and interpret the optimum.
- The existing archive has no D1 pairs, so these original tasks do not establish a reviewed D1 past-paper bank.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Minimum spanning tree**: A connected cycle-free network joining every vertex with the smallest possible total edge weight.
- Kruskal selects edges in increasing weight while avoiding cycles.
- Watch for: A spanning tree has no cycles and joins all vertices.

D1 · Critical paths and scheduling

Must every activity wait for every other?

- A school event has activities that can run in parallel. Adding every duration overestimates the minimum completion time.
- This lesson studies **float** 浮动时间: The time an activity can be delayed without delaying completion under the network model.

The method

- For an activity network, calculate earliest event times forward and latest event times backward.
- Total float for activity i to j is L_j-E_i-duration.
- A critical activity has zero total float; more than one critical path may exist.

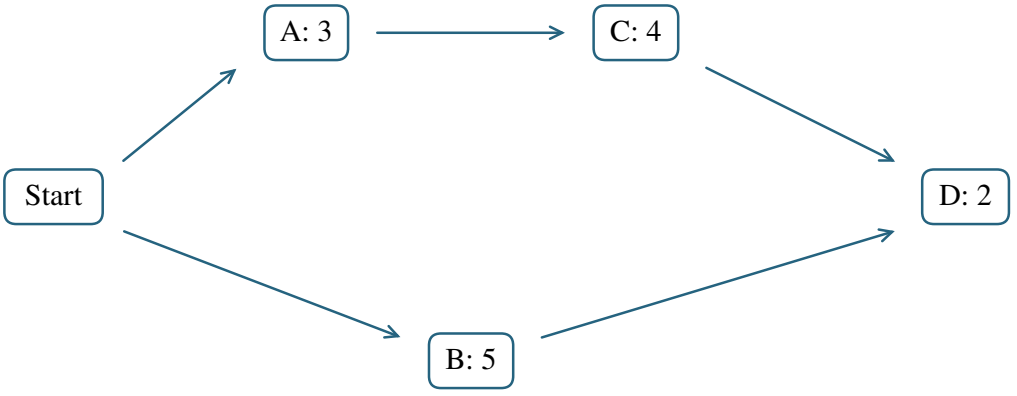
A worked case

$$F_{ij} = L_j - E_i - d_{ij}$$

■ Worked example

Activities A=3 and B=5 start together. C=4 follows A, and D=2 follows both B and C. The earliest completion of C is 7; D must wait until max(5,7)=7 and finishes at 9. Path A-C-D is critical; B has float 2.

Parallel activities meet before D starts



Where it goes wrong

- Do not add durations of independent parallel activities.
- A zero-float activity belongs to a critical path, but there may be several such paths.
- Resource limits can require a schedule longer than the network's theoretical minimum.

> ■ **Watch out**
>
> **Not true:** The minimum project duration is the sum of every activity duration. Activities that run at the same time overlap; the minimum duration is the length of the critical path.

Applying it

- Draw precedence relationships before assigning times.
- Distinguish activity duration from event time.
- Explain what a delay does to the completion date, and record any resource assumptions.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Float**: The time an activity can be delayed without delaying completion under the network model.
- For an activity network, calculate earliest event times forward and latest event times backward.
- Watch for: do not add durations of independent parallel activities.

D1 · Sorting, packing and network algorithms

Is a fast packing method always optimal?

- A packing method quickly fills boxes, but a fast valid arrangement need not use the smallest number of boxes.
- This lesson studies **algorithm** 算法: A finite set of ordered instructions that solves a defined class of problems.

The method

- Trace the named algorithm exactly, including its tie rules.
- In first-fit packing, place each item in the first available bin that can hold it.
- First-fit decreasing sorts before applying first-fit.
- A heuristic may be valid without being optimal.

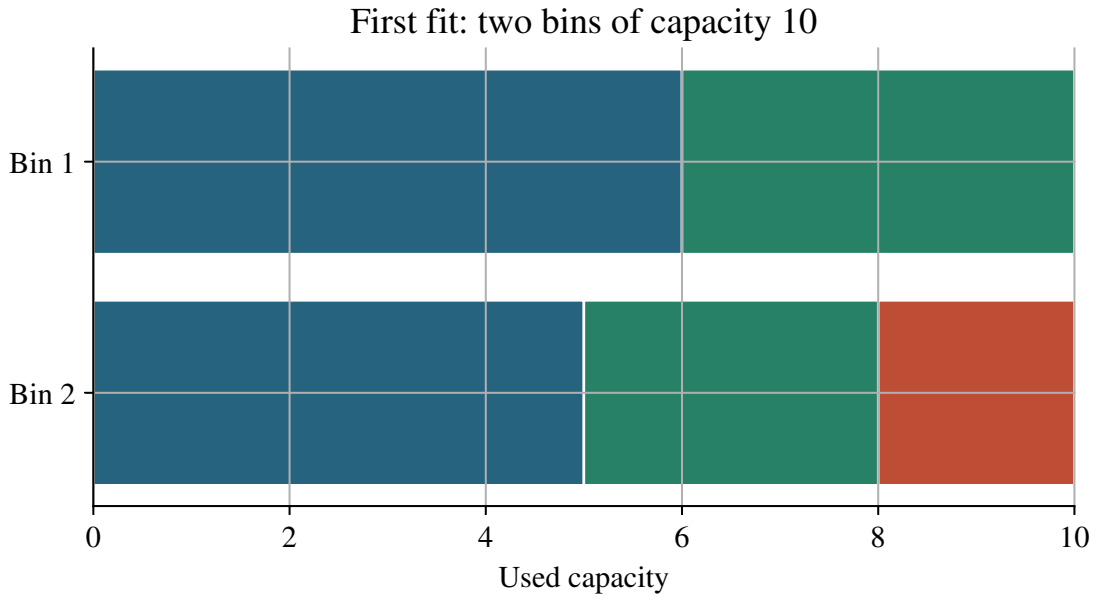
A worked case

$$B_{\min} \geq \left\lceil \frac{\sum s_i}{C} \right\rceil$$

■ Worked example

With bin capacity 10 and items 6,5,4,3,2 in that order, first-fit places 6 and 4 in bin 1, then 5,3,2 in bin 2. It uses 2 bins. The total size is 20, so the lower bound is ceil(20/10)=2; this arrangement is optimal for this instance.

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Where it goes wrong

- An example of success does not prove a heuristic is always optimal.
- Keep intermediate lists in a sorting trace; do not jump from input to a sorted final list.
- A shortest-path update must retain predecessor information if a route is requested.

> ■ **Watch out**
>
> **Not true:** A packing heuristic that works well on one example must be optimal. A heuristic gives a good answer quickly but is not guaranteed to be optimal for every input.

Applying it

- For Dijkstra, choose the smallest unsettled tentative label and update its neighbours.
- For route-inspection problems, distinguish a closed route from an open one and identify odd vertices before pairing them.

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In your course

- Pearson Edexcel International A Level Further Mathematics, unit D1.
- Every paper allows a calculator, but method marks still need the expression or equation written before its value.

■ Key idea

- **Algorithm**: A finite set of ordered instructions that solves a defined class of problems.
- Trace the named algorithm exactly, including its tie rules.
- Watch for: an example of success does not prove a heuristic is always optimal.

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