Probability

AQA GCSE 8300 topic 3.5 full package: three-tier P1-P9 with June 2024 Higher anchors

Probability

- use the **probability scale** and the fact that probabilities of all possible outcomes add up to 1;
- estimate probabilities from experiments and calculate expected frequencies;
- list outcomes systematically with tables, sample spaces and Venn diagrams;

Probability (2)

- use tree diagrams for **independent** and dependent events;
- (higher tier) calculate conditional probabilities from tables, trees and Venn diagrams.

The probability scale and outcomes

- The probability of an event is a number from 0 to 1 on the **probability scale 概率标度**. 0 means impossible, 1 means certain, and 0.5 means an even chance.
- Write probabilities as fractions, decimals or percentages, never as "1 in 4" or "1 : 4".
- When every **outcome 结果** is **equally likely 等可能**, the **theoretical probability 理论概率** is
- For a fair six-sided die, $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

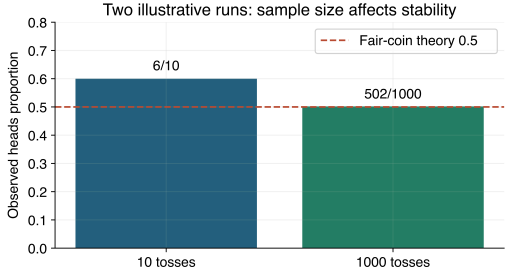
$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$$

Experiments and expected frequency

- When outcomes are not **equally likely**, such as a drawing pin landing point up, we estimate the probability by experiment.
- The **relative frequency 相对频率** is
- A larger **sample size 样本量** gives a better estimate.
- Six heads in 10 tosses gives 0.6, but 502 heads in 1000 tosses gives 0.502, much closer to the true 0.5 of a fair coin.

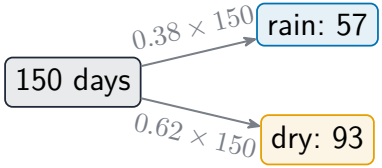
$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

Figure: Experiments and expected frequency



- A small run can be far from 0.5; a large run settles near it: 0.6 against 0.502
- Read the axes and units first.

Figure: Experiments and expected frequency (2)



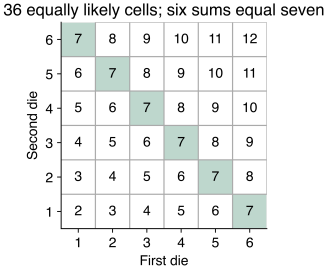
expected counts: a long-run average, not a guarantee

- A **frequency tree** with expected counts: 57 rainy and 93 dry days out of 150
- Match each part of the diagram to the text.

Listing outcomes: sample spaces and Venn diagrams

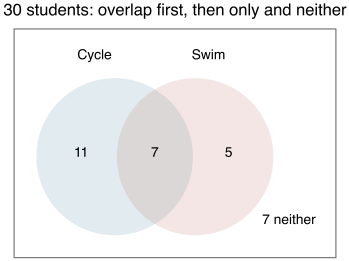
- **Sample spaces.** A **possibility space** 可能性空间, or sample space, lists every **outcome** of a combined experiment.
- Two dice give $6 \times 6 = 36$ **equally likely** outcomes.
- **Worked example 4.** Two fair dice are thrown and the scores added.
- Six of the 36 outcomes give 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).

Figure: Listing outcomes: sample spaces and Venn diagrams



- 36 **equally likely** outcomes; six sum to 7, so $P(7) = 6/36 = 1/6$
- Match each part of the diagram to the text.

Figure: Listing outcomes: sample spaces and Venn diagrams (2)



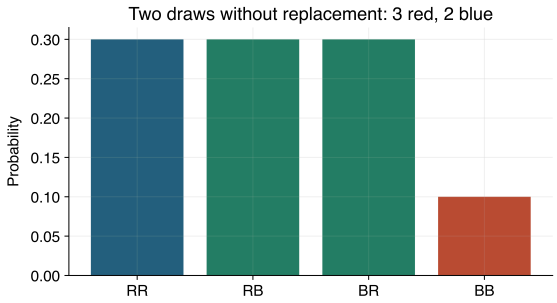
- Fill the overlap first: $11 + 7 + 5 = 23$ do at least one, so 7 do **neither**
- Match each part of the diagram to the text.

Combined events and tree diagrams

- $P(RR) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$;
- $P(BB) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$;
- $P(\text{one of each}) = \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Figure: Combined events and tree diagrams



- **Without replacement**: $P(RR) = 3/5 \times 2/4 = 0.3$, $P(BB) = 2/5 \times 1/4 = 0.1$
- Read the axes and units first.

Conditional probability (H)

- A **conditional probability 条件概率** is the probability of an event when you already know that another event has happened.
- $P(A \mid B)$ means "the probability of A, given B".
- Knowing B shrinks the group you count from to just the members of B:
- **Worked example 9 (H).** In the class of 30 (18 cycle, 12 swim, 7 both), $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$, because only the 18 cyclists count.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Figure: Conditional probability (H)

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

- Given a positive test, use the positive column: 180 out of 260, not 180 out of 200
- Match each part of the diagram to the text.

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.5 (Probability).
- M. vos Savant, "Ask Marilyn", *Parade* (9 September 1990), and her follow-up columns of December 1990 and February 1991.
- AQA, *GCSE Mathematics* Higher Papers 2 and 3 (June 2024), the expected-frequency, dice and machine questions, adapted.

Common errors

- Writing a probability as "1 in 4" or as a ratio.
- Forgetting that the probabilities of all outcomes add up to 1.
- Adding along a branch of a **tree diagram** instead of multiplying.

Common errors (2)

- Keeping the same denominator for the second draw **without replacement**.
- Treating an **expected frequency** as certain.
- Counting the overlap twice in a **Venn diagram**.
- (H) Using the whole group as the denominator of a conditional probability.

Key terms

- **Outcome**: one possible result of an experiment.
- **Mutually exclusive**: events that cannot happen at the same time.
- **Relative frequency**: the number of times an event happened divided by the number of trials.

Key terms (2)

- **Expected frequency**: the number of times an event should happen on average: trials $\times$ probability.
- **Independent**: events where one does not change the probability of the other.
- **Sample space**: a list or table of all possible outcomes.
- **Conditional probability**: the probability of an event given that another event has happened.

Check yourself

- 1.
- A bag has 4 red, 5 green and 1 yellow counter.
- Find $P(\text{green})$ and $P(\text{not red})$. 2.
- A dice is thrown 300 times.