Geometry and measures

AQA GCSE 8300 topic 3.4 full package: three-tier G1-G25 with June 2024 Higher anchors

Geometry and measures

- use angle facts, polygon properties, constructions and loci;
- describe transformations and use **congruence** and similarity;
- calculate perimeters, areas, volumes, arcs and sectors;

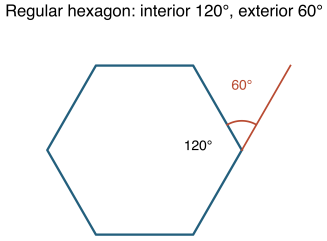
Geometry and measures (2)

- use Pythagoras' theorem and trigonometry, including (higher tier) the sine and cosine rules and 3D problems;
- (higher tier) prove circle theorems and use vectors in proofs.

Angles and polygons

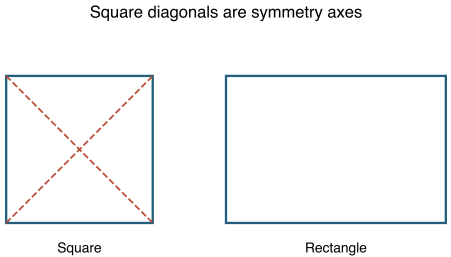
- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- Vertically opposite angles are equal.
- Where a line crosses two parallel lines, alternate angles are equal, **corresponding angles 同位角** are equal, and co-interior angles add up to $180°$. Write "alternate angles are equal", never "Z-angles".
- The angles in a triangle add up to $180°$.

Figure: Angles and polygons



- Regular hexagon: exterior angle $360° \div 6 = 60°$, interior angle $180° - 60° = 120°$
- Match each part of the diagram to the text.

Figure: Angles and polygons (2)

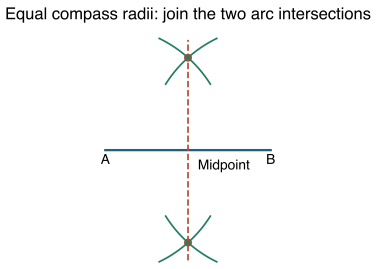


- A square's diagonals are lines of symmetry; a rectangle's are not
- Match each part of the diagram to the text.

Constructions, loci and transformations

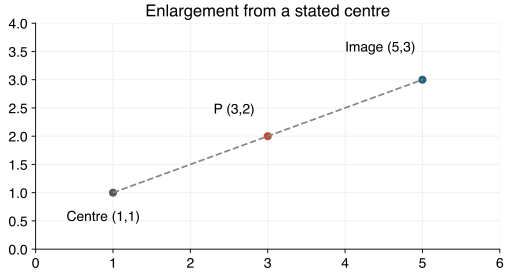
- the **perpendicular bisector 垂直平分线** of a line **segment**, which cuts it in half at right angles;
- the bisector of an angle;
- the perpendicular from a point to a line;
- an angle of $60°$, from an equilateral triangle.

Figure: Constructions, loci and transformations



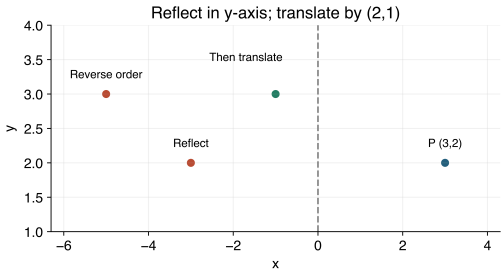
- Equal compass arcs from A and B cross twice; the line through the crossings is the **perpendicular bisector**
- Match each part of the diagram to the text.

Figure: Constructions, loci and transformations (2)



- **Scale factor** 2 from (1, 1): the image is twice as far from the centre as P
- Match each part of the diagram to the text.

Figure: Constructions, loci and transformations (3)

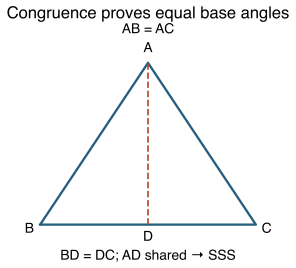


- Order matters: reflect then translate gives $(-1, 3)$; translate then reflect gives $(-5, 3)$
- Read the axes and units first.

Congruence, similarity and circles

- SSS: three sides equal;
- SAS: two sides and the angle between them;
- ASA: two angles and a side in the same position;
- RHS: a right angle, the **hypotenuse** and one other side.

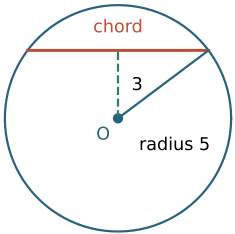
Figure: Congruence, similarity and circles



- AB = AC, BD = DC and AD shared: SSS **congruence**, so the base angles are equal
- Match each part of the diagram to the text.

Figure: Congruence, similarity and circles (2)

Circle of radius 5: distances use one consistent scale

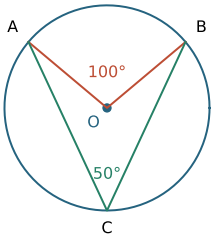


Perpendicular bisects chord: half-length 4

- Radius 5, distance 3: half the chord is $\sqrt{(5^2 - 3^2)} = 4$, so the chord is 8
- Match each part of the diagram to the text.

Figure: Congruence, similarity and circles (3)

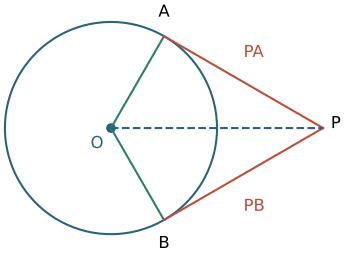
Angles on the same upper arc: centre = twice circumference



- The **angle at the centre**, 100°, is twice the angle at the **circumference**, 50°
- Match each part of the diagram to the text.

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



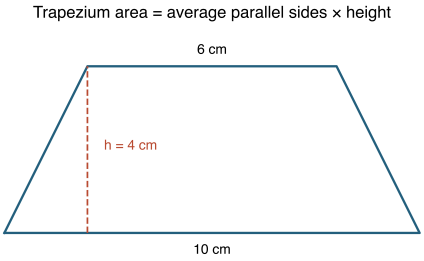
- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height 垂直高度**.
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

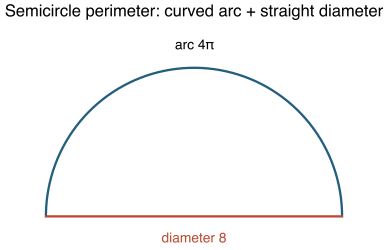
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

Figure: Perimeter, area and volume



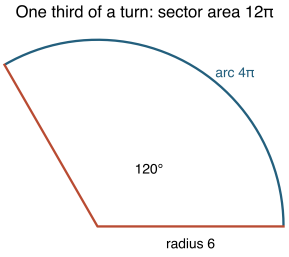
- Area $= \frac{1}{2}(6 + 10) \times 4 = 32$ cm^2
- Match each part of the diagram to the text.

Figure: Perimeter, area and volume (2)



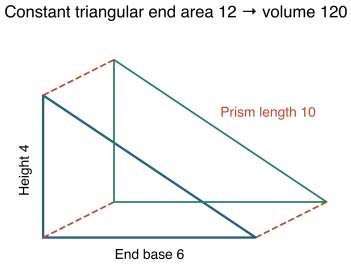
- Perimeter = curved arc + diameter = $4\pi + 8$
- Read the axes and units first.

Figure: Perimeter, area and volume (3)



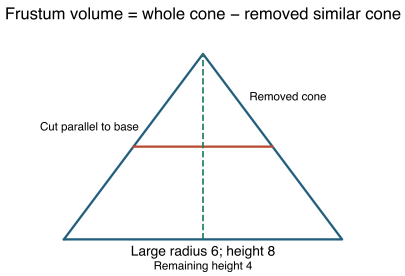
- 120° is one third of a turn: arc $= \frac{1}{3} \times 12\pi = 4\pi$, area $= \frac{1}{3} \times 36\pi = 12\pi$
- Match each part of the diagram to the text.

Figure: Perimeter, area and volume (4)



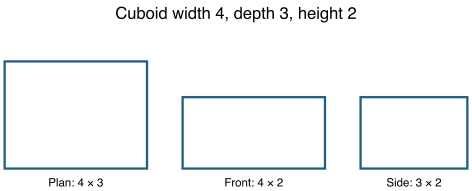
- End area $\frac{1}{2} \times 6 \times 4 = 12$, volume $12 \times 10 = 120$
- Match each part of the diagram to the text.

Figure: Perimeter, area and volume (5)



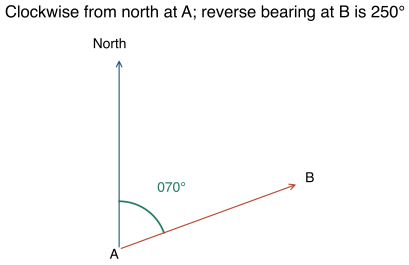
- **Frustum** = whole cone − small cone; the small cone has half the height, so $\frac{1}{8}$ of the volume
- Match each part of the diagram to the text.

Figure: Perimeter, area and volume (6)



- A $4 \times 3 \times 2$ cuboid: each view shows two of its three dimensions
- Match each part of the diagram to the text.

Figure: Perimeter, area and volume (7)



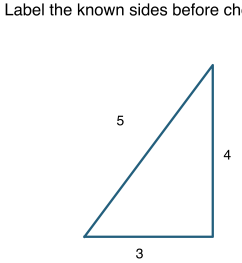
- Bearings run clockwise from north: 070° from A, so the back **bearing** from B is $070° + 180° = 250°$
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse 斜边**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent 邻边** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

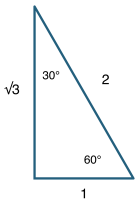
Figure: Pythagoras and trigonometry



- Label opposite, **adjacent** and **hypotenuse** first: 3, 4 and 5 here
- Match each part of the diagram to the text.

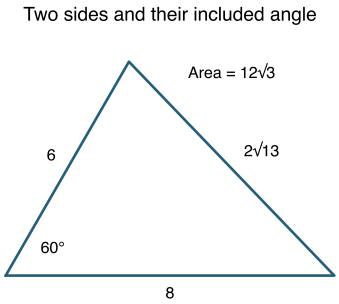
Figure: Pythagoras and trigonometry (2)

Bisected equilateral triangle: sides 1, √3, 2



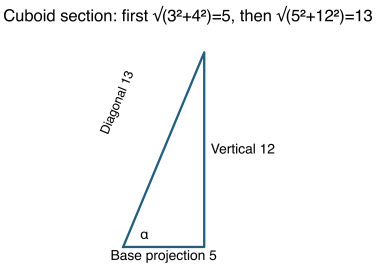
- sin 30° = 1/2, cos 30° = $\sqrt{3}/2$, tan 60° = $\sqrt{3}$ from the 1, $\sqrt{3}$, 2 triangle
- Match each part of the diagram to the text.

Figure: Pythagoras and trigonometry (3)



- Area $= \frac{1}{2} \times 6 \times 8 \times \sin 60^\circ = 12\sqrt{3}$; cosine rule gives the third side $2\sqrt{13}$
- Match each part of the diagram to the text.

Figure: Pythagoras and trigonometry (4)



- Base diagonal $\sqrt{(3^2 + 4^2)} = 5$, then space diagonal $\sqrt{(5^2 + 12^2)} = 13$; tan $\alpha = 12/5$
- Match each part of the diagram to the text.

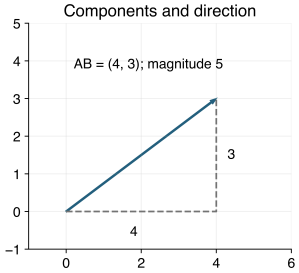
Vectors and coordinate geometry

- **Vectors.** A vector has size and direction.
- The column vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ means 4 right and 3 up; its length is $\sqrt{4^2 + 3^2} = 5$.

Vectors and coordinate geometry (2)

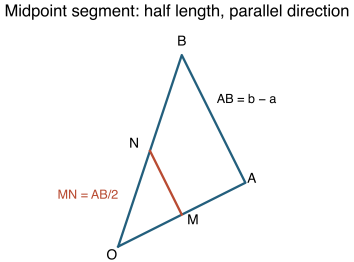
- Add and subtract vectors by adding and subtracting their parts: with $\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.
- **Vector proofs (H).** If one vector is a number times another, the lines are parallel.

Figure: Vectors and coordinate geometry



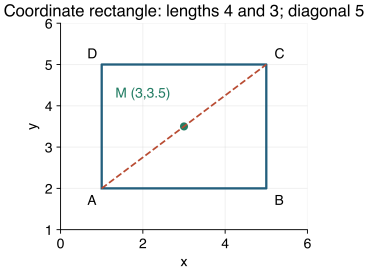
- AB = (4, 3) has magnitude $\sqrt{(4^2 + 3^2)} = 5$
- Match each part of the diagram to the text.

Figure: Vectors and coordinate geometry (2)



- MN = ½b − ½a = ½(b − a) = ½AB: half the length and parallel to AB
- Match each part of the diagram to the text.

Figure: Vectors and coordinate geometry (3)



- Sides 4 and 3 give diagonal 5; the **midpoint** of AC is (3, 3.5)
- Match each part of the diagram to the text.

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.4 (Geometry and measures), and the GCSE formulae sheet.
- Cleomedes, *On the Circular Motions of the Celestial Bodies*, book 1, chapter 7, which records Eratosthenes' method.
- AQA, *GCSE Mathematics* Higher Papers 1 and 2 (June 2024), the trigonometry and vector questions, adapted.

Common errors

- Giving "Z-angles" or "F-angles" as reasons; name alternate or **corresponding angles**.
- Using the slant height instead of the **perpendicular height** in an area or volume formula.
- Forgetting the straight edge when finding the perimeter of a semicircle or **sector**.

Common errors (2)

- Labelling the **hypotenuse** as the **adjacent** side.
- Using the length **scale factor** for an area or volume.
- Rubbing out construction arcs.

Common errors (3)

- Taking only the calculator's acute angle when the sine rule allows two.
- Writing a **bearing** with fewer than three figures, such as $70°$ for $070°$.

Key terms

- **Regular polygon**: a polygon with all sides and all angles equal.
- **Perpendicular bisector**: the line that cuts a **segment** in half at right angles.
- **Locus**: the set of points that follow a given rule.

Key terms (2)

- **Congruent**: exactly the same shape and size.
- **Scale factor**: the number every length is multiplied by in an **enlargement** or between similar shapes.
- **Sector**: the part of a circle between two radii and an arc.

Key terms (3)

- **Hypotenuse**: the longest side of a right-angled triangle, opposite the right angle.
- **Bearing**: a direction measured clockwise from north, written with three figures.
- **Collinear**: lying on one straight line.

Check yourself

- 1.
- Find the sum of the interior angles of an octagon, and the interior angle of a regular octagon. 2.
- Find the area of a circle of diameter 10 cm, in terms of π. 3.
- A **sector** has radius 9 cm and angle $40°$.