Ratio, proportion and rates of change

AQA GCSE 8300 topic 3.3 full package: three-tier R1-R16 with June 2024 Higher anchors

Ratio, proportion and rates of change

- convert between units, including compound units such as speed, **density** and **pressure**;
- simplify ratios, share in a ratio and solve best-buy problems;
- use percentage multipliers, including reverse percentages and **compound interest**;

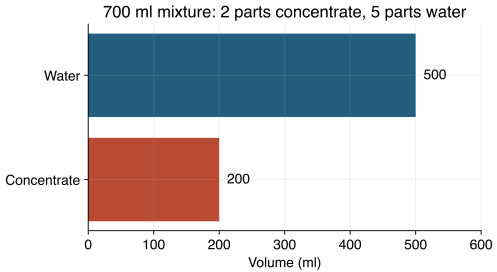
Ratio, proportion and rates of change (2)

- solve direct and **inverse proportion** problems and (higher tier) form their equations;
- use **scale** factors for lengths, areas and volumes, and read a gradient as a **rate of change**.

Ratio and sharing

- A ratio compares two or more quantities.
- Simplify it by dividing every part by the same number: $45 : 60 = 3 : 4$.
- A ratio can also be written as a fraction: in a class with boys and girls in the ratio $3 : 4$, the boys are $\frac{3}{7}$ of the class, because there are 7 parts in the whole.
- This is a **part-to-whole ratio 部分与整体的比**.

Figure: Ratio and sharing



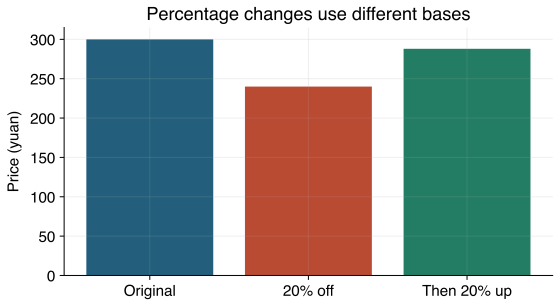
- Ratio 2 : 5 has 7 parts; $700 \div 7 = 100$ ml per part, so 200 ml and 500 ml
- Read the axes and units first.

Percentages

- an increase of 12% means $\times 1.12$;
- a decrease of 17% means $\times 0.83$;
- 120% of an amount means $\times 1.2$.

$$\text{percentage change} = \frac{\text{change}}{\text{original}} \times 100$$

Figure: Percentages

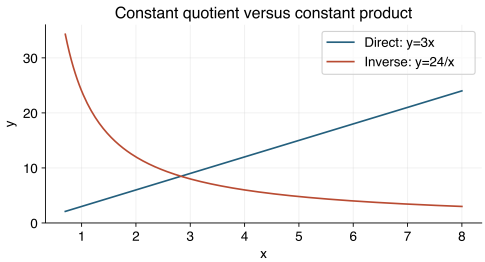


- 20% off then 20% up gives 288, not 300: the second 20% is of 240
- Read the axes and units first.

Direct and inverse proportion

- Two quantities are in **direct proportion** 正比例 when they increase at the same rate: double one and the other doubles.
- We write $y \propto x$, which means $y = kx$.
- The number k is the **constant of proportionality** 比例常数.
- The graph is a straight line through the origin.

Figure: Direct and inverse proportion



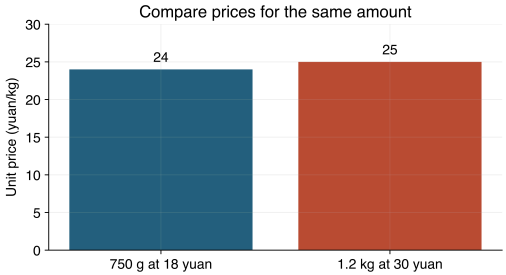
- **Direct proportion** keeps y ÷ x constant (3); **inverse proportion** keeps x × y constant (24)
- Read the axes and units first.

Units and compound measures

- **Converting units.** Learn the metric links: 1 km $=$ 1000 m, 1 m $=$ 100 cm, 1 kg $=$ 1000 g, 1 litre $=$ 1000 cm^3.
- Time is not decimal: 2 hours 36 minutes is $2 + \frac{36}{60} = 2.6$ hours, not 2.36 hours.
- A compound unit combines two units, such as km/h or g/cm^3.
- **Density** 密度 tells you how much mass is packed into each unit of volume. **Pressure** 压强 is the force on each unit of area, measured in N/m^2.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{density} = \frac{\text{mass}}{\text{volume}} \qquad \text{pressure} = \frac{\text{force}}{\text{area}}$$

Figure: Units and compound measures

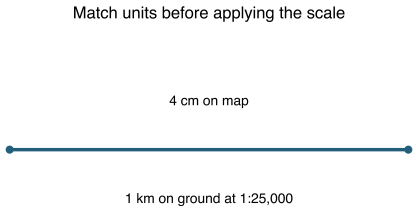


- Compare the same amount: $18 \div 0.75 = 24$ yuan/kg beats $30 \div 1.2 = 25$ yuan/kg
- Read the axes and units first.

Scale, similarity and rates of change

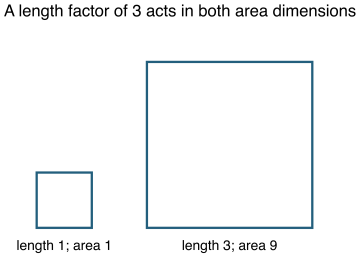
- **Scale drawings.** A map **scale** 比例尺 of $1 : 25\,000$ means 1 cm on the map is 25 000 cm in real life.
- A 4 cm line on the map is $4 \times 25\,000 = 100\,000$ cm, which is 1 km.
- **Length, area and volume.** If two shapes are similar with a length **scale** factor of k, the **area scale factor** 面积比例因子 is k^2 and the volume scale factor is k^3.
- A square with sides 3 times as long has 9 times the area.

Figure: Scale, similarity and rates of change



- $4\ \text{cm} \times 25\ 000 = 100\ 000\ \text{cm} = 1\ \text{km}$
- Match each part of the diagram to the text.

Figure: Scale, similarity and rates of change ·



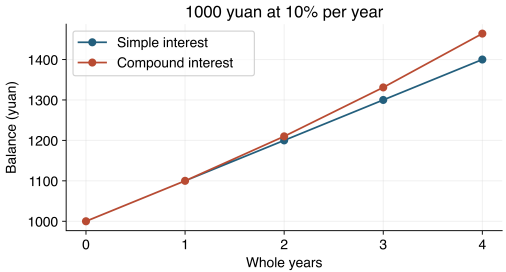
- A length factor of 3 multiplies area by $3 \times 3 = 9$
- Match each part of the diagram to the text.

Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest 复利** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where P is the starting amount, r the percentage rate and n the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

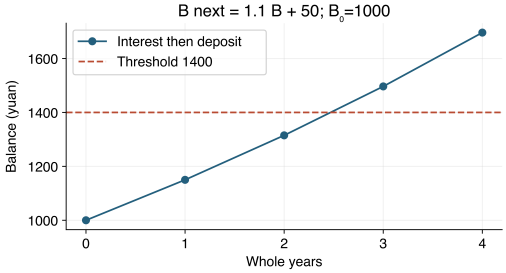
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Figure: Growth and decay



- Simple interest adds 100 each year; **compound interest** grows by 10% of the new balance
- Read the axes and units first.

Figure: Growth and decay ·



- $B_{n+1} = 1.1B_n + 50$: 1000, 1150, 1315, 1496.5, so it first passes 1400 after 3 years
- Match each part of the diagram to the text.

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.3 (Ratio, proportion and rates of change).
- NASA, *Mars Climate Orbiter Mishap Investigation Board Phase I Report* (10 November 1999).
- AQA, *GCSE Mathematics* Higher Paper 2 (June 2024), the reverse-percentage and house-price questions, adapted.

Common errors

- Sharing in the ratio $2:5$ by dividing by 5 instead of by the total number of parts, 7.
- Adding the percentage back on in a reverse percentage; divide by the **multiplier** instead.
- Thinking a 20% fall followed by a 20% rise returns to the start.

Common errors (2)

- Writing 2 hours 36 minutes as 2.36 hours.
- Using the length **scale** factor for an area or a volume.
- Mixing units, such as kg and cm^3, when finding **density**.
- Using simple interest when a question says "compound".

Key terms

- **Ratio**: a comparison of the sizes of two or more quantities, such as $3 : 4$.
- **Multiplier**: the number you multiply by to make a percentage change.
- **Direct proportion**: $y = kx$; both quantities change by the same factor.

Key terms (2)

- **Inverse proportion**: $y = \frac{k}{x}$; when one doubles, the other halves.
- **Density**: mass divided by volume.
- **Pressure**: force divided by area.

Key terms (3)

- **Compound interest**: interest added to the amount each year, so that later interest is paid on it too.
- **Rate of change**: how fast one quantity changes compared with another; the gradient of a graph.

Check yourself

- 1.
- Share 560 yuan in the ratio $3:5$. 2.
- A 750 g pack costs 18 yuan and a 1.2 kg pack costs 30 yuan.
- Which is better value? 3.