

Ratio, proportion and rates of change – native teaching supplement

AQA 8300: Foundation and Higher sections retain their labelled scope

1. Supported teaching and tier boundary

Supported teaching and tier boundary

8300: Ratio, proportion and rates of change. Version: Version 1.0, 12 September 2014; first examination 2017.

2. Supported teaching and tier boundary

Supported teaching and tier boundary

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

3. Percentages, ratio and proportional reasoning · Foundation

Percentages, ratio and proportional reasoning · Foundation

A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.

4. Percentages, ratio and proportional reasoning · Foundation

Percentages, ratio and proportional reasoning · Foundation

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

5. Percentages, ratio and proportional reasoning · Foundation

Percentages, ratio and proportional reasoning · Foundation

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.

6. Percentages, ratio and proportional reasoning · Foundation

Percentages, ratio and proportional reasoning · Foundation

A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.

7. Percentages, ratio and proportional reasoning · Foundation

Percentages, ratio and proportional reasoning · Foundation

Use percentage multipliers and divide a total into ratio parts. This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.

8. Percentages, ratio and proportional reasoning · Higher

Percentages, ratio and proportional reasoning · Higher

A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.

9. Percentages, ratio and proportional reasoning · Higher

Percentages, ratio and proportional reasoning · Higher

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

10. Percentages, ratio and proportional reasoning · Higher

Percentages, ratio and proportional reasoning · Higher

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240 / 0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.

11. Percentages, ratio and proportional reasoning · Higher

Percentages, ratio and proportional reasoning · Higher

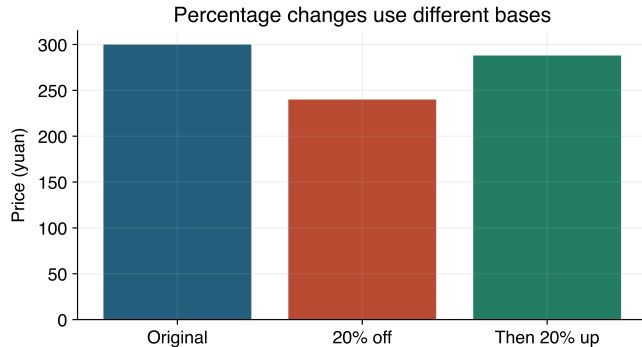
A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.

12. Percentages, ratio and proportional reasoning · Higher

Percentages, ratio and proportional reasoning · Higher

For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant.

Figure 13: Percentages, ratio



Check the labelled values and conditions.

14. Ratios, equivalent proportions and mixtures · Foundation

Ratios, equivalent proportions and mixtures · Foundation

Put quantities in the same units before simplifying a ratio. Divide every part by the same nonzero factor. For $a:b$ the whole has $a+b$ parts; the first share is $a/(a+b)$ of the whole but a/b of the other share. Equivalent ratios have the same multiplicative relationship: $a/b=c/d$ gives $ad=bc$ when denominators are nonzero.

15. Ratios, equivalent proportions and mixtures · Foundation

Ratios, equivalent proportions and mixtures · Foundation

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc, \quad C = \frac{2}{7}V, \quad W = \frac{5}{2}C$$

16. Ratios, equivalent proportions and mixtures · Foundation

Ratios, equivalent proportions and mixtures · Foundation

For 700 ml at 2:5, each part is 100 ml: concentrate 200 ml and water 500 ml.

Concentrate:whole=2:7, while concentrate/water=2/5. Water/concentrate=5/2, a fraction greater than 1. Doubling both gives 400:1000=2:5. If concentrate is x and water y , the recipe requires $y=(5/2)x$, a straight line through the origin. For 300 ml concentrate, water is 750 ml and the total is 1050 ml. A batch with 200 ml concentrate and 600 ml water has ratio 1:3 and is weaker. To simplify 1.5 litres:500 ml, first write $1500:500=3:1$.

17. Ratios, equivalent proportions and mixtures · Foundation

Ratios, equivalent proportions and mixtures · Foundation

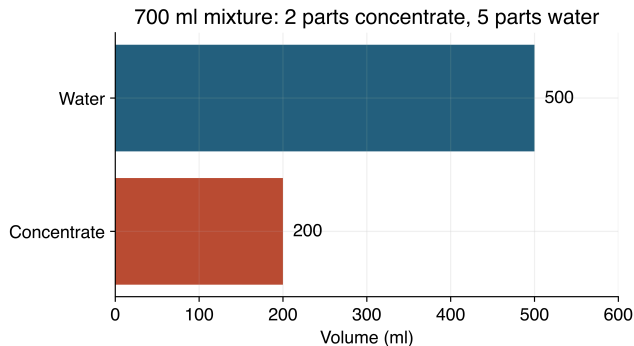
Adding the same amount to both shares generally changes the ratio. Do not divide by $2+7$ when the stated ratio is already part:whole 2:7; the whole is seven parts. A concentration fraction uses total volume as denominator.

18. Ratios, equivalent proportions and mixtures · Foundation

Ratios, equivalent proportions and mixtures · Foundation

AQA R3–R8 connects ratio notation, fractions, equivalent proportions and linear functions. Check that both shares sum to the stated total, and name which quantity is compared with which. Use a ratio table to scale a mixture without assuming a fixed additive difference.

Figure 19: Ratios, equivalent proportions



Check the labelled values and conditions.

20. Scale drawings and maps · Foundation

Scale drawings and maps · Foundation

In a scale 1:n, one drawing unit represents n of the same real unit. Multiply a drawing length by n to obtain the real length; divide a real length by n to draw it. Then convert the unit. Measure only when the diagram explicitly supplies an accurate scale.

21. Scale drawings and maps · Foundation

Scale drawings and maps · Foundation

$$L_{\text{real}} = nL_{\text{drawing}}$$

22. Scale drawings and maps · Foundation

Scale drawings and maps · Foundation

At 1:25,000, 4 cm represents $100,000 \text{ cm} = 1000 \text{ m} = 1 \text{ km}$. A real 1.5 km path is 150,000 cm, so it measures $150,000 / 25,000 = 6 \text{ cm}$ on the map. A room 6 m by 4 m drawn at 1:100 becomes 6 cm by 4 cm because each metre is 100 cm. Its drawing diagonal is $\sqrt{(6^2 + 4^2)} \approx 7.21 \text{ cm}$ and the real diagonal is about 7.21 m. Enlarging the printed map changes its numerical scale: doubling drawing lengths halves the scale denominator. A scale bar printed with the map enlarges with it.

23. Scale drawings and maps · Foundation

Scale drawings and maps · Foundation

A ratio compares matching units. Never measure a diagram marked not to scale. A photocopied numerical scale can become invalid even though its scale bar still works.

24. Scale drawings and maps · Foundation

Scale drawings and maps · Foundation

AQA R2 includes maps, scale factors and geometric problems. Label drawing and real dimensions separately; reverse the calculation to verify the scale. Use an exact ratio until the context asks for rounding.

Figure 25: Scale drawings and maps

Match units before applying the scale

4 cm on map



1 km on ground at 1:25,000

Check the labelled values and conditions.

26. Direct and inverse proportional relationships · Foundation

Direct and inverse proportional relationships · Foundation

Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin. Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded. State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion. Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

27. Direct and inverse proportional relationships · Foundation

Direct and inverse proportional relationships · Foundation

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

28. Direct and inverse proportional relationships · Foundation

Direct and inverse proportional relationships · Foundation

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. The inverse model assumes equally productive workers on one fixed job.

29. Direct and inverse proportional relationships · Foundation

Direct and inverse proportional relationships · Foundation

An increasing graph need not be direct proportion. An inverse relationship is proportional to $1/x$, not to $-x$. Keep a product constant for inverse proportion and a quotient constant for direct proportion.

30. Direct and inverse proportional relationships · Foundation

Direct and inverse proportional relationships · Foundation

AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion. Interpret the units of k and the gradient. In Foundation use the supplied equations; Higher also constructs and interprets them from the context.

31. Direct and inverse proportional relationships · Higher

Direct and inverse proportional relationships · Higher

Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin. Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded. State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion. Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

32. Direct and inverse proportional relationships · Higher

Direct and inverse proportional relationships · Higher

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

33. Direct and inverse proportional relationships · Higher

Direct and inverse proportional relationships · Higher

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. Higher construction: if y is directly proportional to x and $y=18$ at $x=6$, $k=18/6=3$, hence $y=3x$. If t is inversely proportional to w and $t=6$ at $w=4$, $k=tw=24$, hence $t=24/w$. These models assume a constant rate and exclude impossible negative worker counts.

34. Direct and inverse proportional relationships · Higher

Direct and inverse proportional relationships · Higher

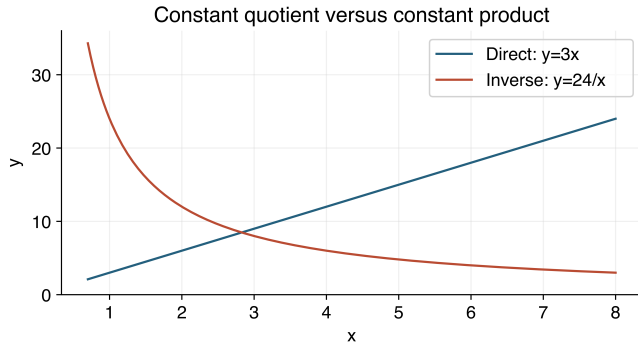
An increasing graph need not be direct proportion. An inverse relationship is proportional to $1/x$, not to $-x$. Keep a product constant for inverse proportion and a quotient constant for direct proportion.

35. Direct and inverse proportional relationships · Higher

Direct and inverse proportional relationships · Higher

AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion. Interpret the units of k and the gradient. In Foundation use the supplied equations; Higher also constructs and interprets them from the context.

Figure 36: Direct and inverse proportional



Check the labelled values and conditions.

37. Rates, unit prices, density and pressure · Foundation

Rates, unit prices, density and pressure · Foundation

A compound rate divides one quantity by another: $\text{speed} = d/t$, $\text{pay} = \text{earnings}/\text{time}$, $\text{unit price} = \text{cost}/\text{amount}$, $\text{density} = \text{mass}/\text{volume}$ and $\text{pressure} = \text{force}/\text{area}$. Rearrange these equations before substituting. Convert each dimension separately; converting cm^2 to m^2 uses a squared length factor. Comparisons must use the same units and conditions.

38. Rates, unit prices, density and pressure · Foundation

Rates, unit prices, density and pressure · Foundation

$$\rho = \frac{m}{V}, \quad p = \frac{F}{A}, \quad m = \rho V$$

39. Rates, unit prices, density and pressure · Foundation

Rates, unit prices, density and pressure · Foundation

A 750 g pack costing 18 yuan has unit price $18/0.75=24$ yuan/kg. A 1.2 kg pack at 30 yuan costs 25 yuan/kg, so the first is better value if quality and waste are equal. A block of mass 540 g and volume 200 cm^3 has density 2.7 g/cm^3 ; a 50 cm^3 piece of that material has mass 135 g. Since $1\text{ g}=0.001\text{ kg}$ and $1\text{ cm}^3=0.000001\text{ m}^3$, $2.7\text{ g/cm}^3=2700\text{ kg/m}^3$. A 120 N force over 0.03 m^2 gives pressure $4000\text{ N/m}^2=4000\text{ Pa}$. At fixed force, halving the area doubles pressure. An hourly pay rate of 48 yuan/hour gives 120 yuan for 2.5 hours.

40. Rates, unit prices, density and pressure · Foundation

Rates, unit prices, density and pressure · Foundation

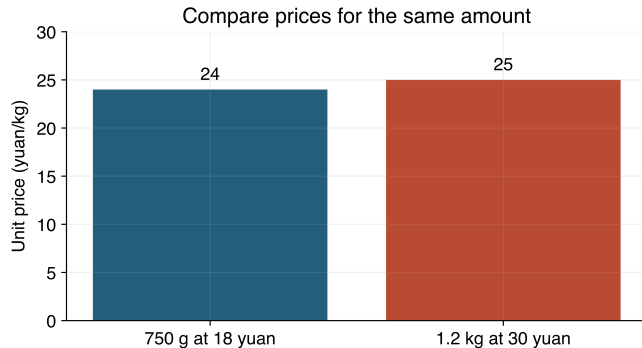
Volume conversions cube the length factor and area conversions square it. A density of 2.7 g/cm^3 is not 2.7 kg/m^3 . A cheaper package may cost more per kilogram; include only comparable products.

41. Rates, unit prices, density and pressure · Foundation

Rates, unit prices, density and pressure · Foundation

AQA R1/R11 includes speed, pay, pricing, density and pressure in numerical and algebraic contexts. Show the rearrangement with named quantities and carry units through the answer; use an inverse check such as $\text{density} \times \text{volume} = \text{mass}$.

Figure 42: Rates, unit prices, density



Check the labelled values and conditions.

43. Length, area and volume scale factors · Foundation

Length, area and volume scale factors · Foundation

For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 . Name the direction of the comparison: model to real or real to model. Recover k from an area ratio by a square root and from a volume ratio by a cube root. A change in only one dimension does not create similar solids. Higher links corresponding sides to equal trigonometric ratios.

44. Length, area and volume scale factors · Foundation

Length, area and volume scale factors · Foundation

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

45. Length, area and volume scale factors · Foundation

Length, area and volume scale factors · Foundation

Enlarging a box from $2 \times 3 \times 4$ to $6 \times 9 \times 12$ multiplies lengths by 3. Its volume changes from 24 to 648, a factor of 27; a 2×3 face changes area from 6 to 54, a factor of 9. Length ratio 2:5 corresponds to area ratio 4:25 and volume ratio 8:125. If similar shapes have areas 20 and 80 cm^2 , the larger-to-smaller length factor is $\sqrt{(80/20)}=2$. If similar solids have volumes 16 and 128 cm^3 , $k=\sqrt[3]{8}=2$.

46. Length, area and volume scale factors · Foundation

Length, area and volume scale factors · Foundation

Areas do not scale by k , and volumes do not scale by k^2 . Similarity needs every corresponding length to share the same factor. Reversing a ratio requires the reciprocal factor.

47. Length, area and volume scale factors · Foundation

Length, area and volume scale factors · Foundation

AQA R12 Foundation includes ratios and scale factors for lengths, areas and volumes. Higher adds links to similarity including trigonometric ratios. Check dimensions and compare a simple box or rectangle before applying the general factor.

48. Length, area and volume scale factors · Higher

Length, area and volume scale factors · Higher

For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 . Name the direction of the comparison: model to real or real to model. Recover k from an area ratio by a square root and from a volume ratio by a cube root. A change in only one dimension does not create similar solids. Higher links corresponding sides to equal trigonometric ratios.

49. Length, area and volume scale factors · Higher

Length, area and volume scale factors · Higher

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

50. Length, area and volume scale factors · Higher

Length, area and volume scale factors · Higher

Enlarging a box from $2 \times 3 \times 4$ to $6 \times 9 \times 12$ multiplies lengths by 3. Its volume changes from 24 to 648, a factor of 27; a 2×3 face changes area from 6 to 54, a factor of 9. Length ratio 2:5 corresponds to area ratio 4:25 and volume ratio 8:125. If similar shapes have areas 20 and 80 cm^2 , the larger-to-smaller length factor is $\sqrt{(80/20)}=2$. If similar solids have volumes 16 and 128 cm^3 , $k=\sqrt[3]{8}=2$. Higher: triangles with the same acute angle have equal opposite/hypotenuse ratios; doubling both lengths leaves $\sin\theta$ unchanged.

51. Length, area and volume scale factors · Higher

Length, area and volume scale factors · Higher

Areas do not scale by k , and volumes do not scale by k^2 . Similarity needs every corresponding length to share the same factor. Reversing a ratio requires the reciprocal factor.

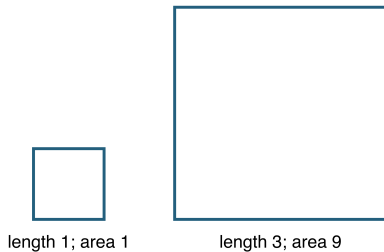
52. Length, area and volume scale factors · Higher

Length, area and volume scale factors · Higher

AQA R12 Foundation includes ratios and scale factors for lengths, areas and volumes. Higher adds links to similarity including trigonometric ratios. Check dimensions and compare a simple box or rectangle before applying the general factor.

Figure 53: Length, area and volume scale

A length factor of 3 acts in both area dimensions



Check the labelled values and conditions.

54. Simple interest, compound growth and decay · Foundation

Simple interest, compound growth and decay · Foundation

A percentage r corresponds to decimal $r/100$; increases use multiplier $1+r/100$ and decreases $1-r/100$. Simple interest on principal P for n periods at rate r is $Pn/100$, giving balance $P(1+rn/100)$. Compound balance is $P(1+r/100)^n$. Decay uses the corresponding decreasing multiplier. Use matching rate and period units, and divide by the multiplier to recover an original value.

55. Simple interest, compound growth and decay · Foundation

Simple interest, compound growth and decay · Foundation

$$B_n = P \left(1 + \frac{r}{100} \right)^n$$

56. Simple interest, compound growth and decay · Foundation

Simple interest, compound growth and decay · Foundation

At 10% per year, 1000 yuan earns simple interest 100 per year: after two years interest is 200 and balance is 1200. Compound balances are 1100 after year one and 1210 after year two. A machine worth 800 yuan losing 20% each year becomes 640 then 512, not 480: the second loss is 20% of 640. An 800-yuan sale price after a 20% reduction corresponds to original price $800/0.8=1000$. Growth from 50 to 65 is $(65-50)/50 \times 100 = 30\%$; 65 is 130% of 50. If a compound balance must first exceed 1300 at 10%, year two gives 1210 and year three 1331, so three whole years are needed.

57. Simple interest, compound growth and decay · Foundation

Simple interest, compound growth and decay · Foundation

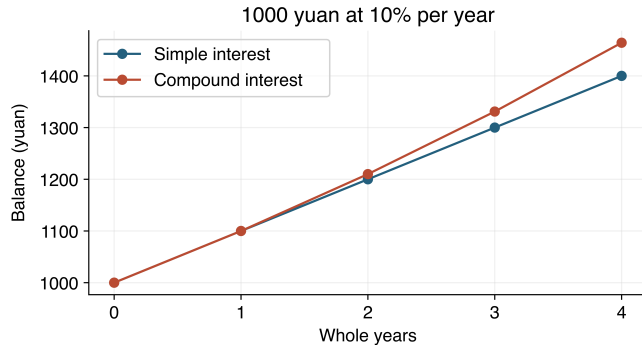
Use the original value as the denominator for percentage change. Repeated 20% decreases do not subtract 40% of the initial amount. A rate per year cannot be treated as a rate per month without a specified conversion.

58. Simple interest, compound growth and decay · Foundation

Simple interest, compound growth and decay · Foundation

AQA R9/R16 includes percentage comparisons, original values, simple interest and repeated growth/decay. State whether the task asks for interest alone or total balance, and interpret whole-period threshold answers.

Figure 59: Simple interest, compound growth



Check the labelled values and conditions.

60. Iterative processes with repeated deposits · Higher

Iterative processes with repeated deposits · Higher

Write an update rule and an initial value. Apply the operations in their stated order, using the previous output as the next input. A recurrence $B_{\text{next}} = 1.1B + 50$ differs from $1.1(B + 50)$. Tables can locate a first whole-period threshold; verify both the preceding and crossing values. Iteration can model growth, decay or other repeated processes, not just solve equations.

61. Iterative processes with repeated deposits · Higher

Iterative processes with repeated deposits · Higher

$$B_{n+1} = 1.1B_n + 50, \quad B_0 = 1000$$

62. Iterative processes with repeated deposits · Higher

Iterative processes with repeated deposits · Higher

With $B_0=1000$ and $B_{n+1}=1.1B_n+50$, $B_1=1150$, $B_2=1315$ and $B_3=1496.5$. The balance first exceeds 1400 after three years because $1315 \leq 1400 < 1496.5$. If the deposit preceded interest, $B_1=1.1 \times 1050=1155$, five yuan greater. A decay-and-top-up rule $V_{\text{next}}=0.8V+20$, starting at 200, gives 180 then 164. A fixed point satisfies $V=0.8V+20$, hence $V=100$; values above 100 decrease toward it. A fixed point is a value preserved by the update, not a claim that every finite step reaches it exactly.

63. Iterative processes with repeated deposits · Higher

Iterative processes with repeated deposits · Higher

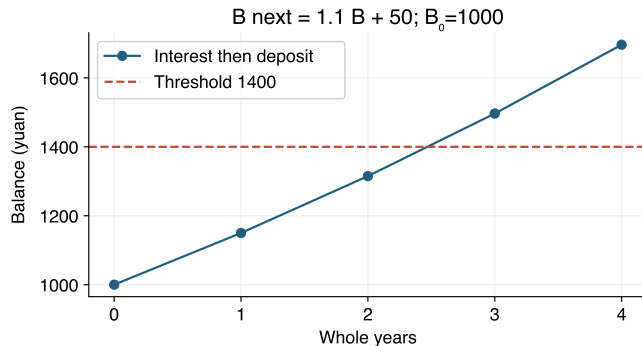
Preserve operation order and use the updated value each time. Do not round early or confuse the initial value with the first updated value. A continuous fractional-period estimate does not answer a whole-period question by itself.

64. Iterative processes with repeated deposits · Higher

Iterative processes with repeated deposits · Higher

AQA R16 Higher extends growth/decay to general iterative processes. State the model assumptions, initial value, recurrence and threshold interpretation. Use substitution to check a proposed fixed point without calculus.

Figure 65: Iterative processes with repeated



Check the labelled values and conditions.

66. Try independently

Try independently

A 1:25000 map shows a road as 6.4 cm. Find its real length in kilometres. A cyclist travels it in eight minutes. Find average speed in kilometres per hour, showing the unit conversion.

67. Reasoning and check

Reasoning and check

Real length $L = kl = 25000(6.4 \text{ cm}) = 160000 \text{ cm} = 1.6 \text{ km}$. Time $t = 8/60 \text{ h} = 2/15 \text{ h}$. Average speed $v = L/t = (1.6 \text{ km})/(2/15 \text{ h}) = 12 \text{ km/h}$. The map ratio compares like units before conversion. The average does not assert constant instantaneous speed.

68. Try independently

Try independently

A savings account starts at 1000 CNY. Each year it receives 5% interest, then a 100 CNY deposit. Write a recurrence and find the balance after two years. Compare with depositing before interest each year, explaining the difference.

69. Reasoning and check

Reasoning and check

Interest then deposit gives $B_{n+1} = 1.05B_n + 100$, with $B_0 = 1000$. Thus $B_1 = 1150$ and $B_2 = 1307.50$ CNY. Deposit first gives $C_{n+1} = 1.05(C_n + 100)$, so $C_1 = 1155$ and $C_2 = 1317.75$ CNY. The difference is 10.25 CNY: the first early deposit earns five CNY an extra year and that advantage earns 5% in year two, while the second early deposit adds another five CNY. Operation order is part of the model.

70. Terms

Terms	
multiplier	.

71. Terms

Terms

part-to-whole ratio

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72. Terms

Terms	
scale	.

73. Terms

Terms

constant of proportionality .

74. Terms

Terms
density .

75. Terms

Terms

area scale factor

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76. Terms

Terms

compound interest .

77. Terms

Terms

recurrence relation .

Check yourself

Reconstruct the method without looking back

A savings account starts at 1000 CNY. Each year it receives 5% interest, then a 100 CNY deposit. Write a recurrence and find the balance after two years. Compare with depositing before interest each year, explaining the difference.