Algebra

AQA GCSE 8300 topic 3.2 full package: three-tier A1-A25 with June 2024 Higher anchors

Algebra: letters that stand for numbers

- use algebraic notation, expand brackets and factorise expressions;
- rearrange formulae and (higher tier) prove identities and use function notation;
- draw and read straight-line, quadratic and other graphs;

Algebra: letters that stand for numbers (2)

- solve linear and quadratic equations, simultaneous equations and inequalities;
- find and use the **nth term** of a sequence.

Expressions, formulae and identities

- An expression has no equals sign: $4x - 1$.
- An equation is true for particular values: $4x - 1 = 11$ is true only when $x = 3$.
- A formula connects quantities: $v = u + at$.
- An **identity** **恒等式** is true for every value of the letter, and uses the sign $\equiv$: $(x + 2)^2 \equiv x^2 + 4x + 4$.

Functions (H)

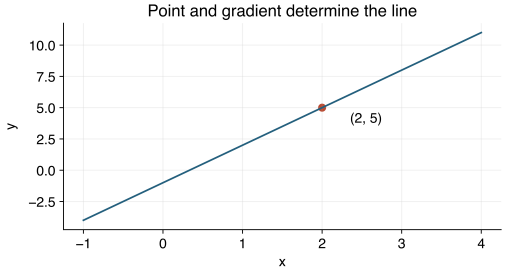
- $fg(2) = f(g(2)) = f(4) = 7$, but $gf(2) = g(f(2)) = g(1) = 1$: the order matters.
- For f^{-1}, write $y = 3x - 5$ and make x the subject: $x = \frac{y+5}{3}$. So $f^{-1}(x) = \frac{x+5}{3}$.
- Check: $f^{-1}(f(4)) = f^{-1}(7) = \frac{12}{3} = 4$.

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient** 梯度, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

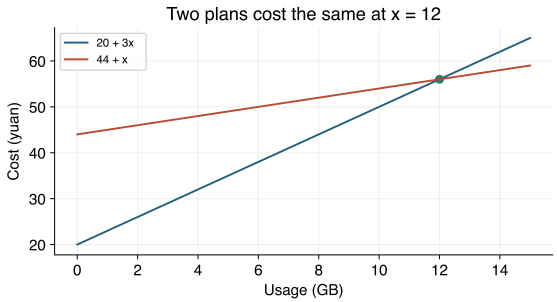
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Figure: Straight-line graphs



- **Gradient** 3 through (2, 5): $y - 5 = 3(x - 2)$, so $y = 3x - 1$
- Match each part of the diagram to the text.

Figure: Straight-line graphs ·



- The plans cost the same where $20 + 3x = 44 + x$: $x = 12$, cost 56 yuan
- Match each part of the diagram to the text.

Quadratic and other graphs

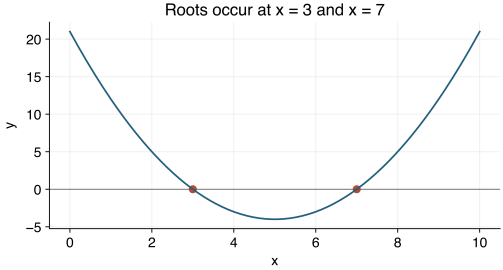
- Cubic, $y = x^3$: an S-shape.
- **Reciprocal 倒数** graph, $y = \frac{1}{x}$: two separate curves; $x = 0$ is not allowed.
- (H) Exponential, $y = 2^x$: grows faster and faster and never touches the x-axis.
- (H) $y = \sin x$, $y = \cos x$ and $y = \tan x$: waves that repeat every $360°$ ($180°$ for $\tan$).

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Figure: Solving equations

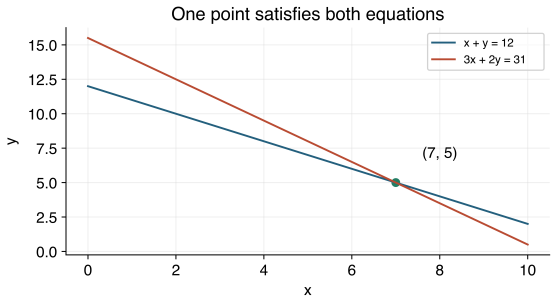


- $x^2 - 10x + 21 = (x - 3)(x - 7)$: roots $x = 3$ and $x = 7$
- Read the axes and units first.

Simultaneous equations and iteration

- Two equations with two unknowns are solved together.
- On a graph, the solution is the point where the two lines cross.
- **Elimination.** In **elimination** 消元法 you add or subtract the equations to remove one letter.
- **Worked example 12.** Solve $3x + 2y = 16$ and $5x - 2y = 8$.

Figure: Simultaneous equations and iteration

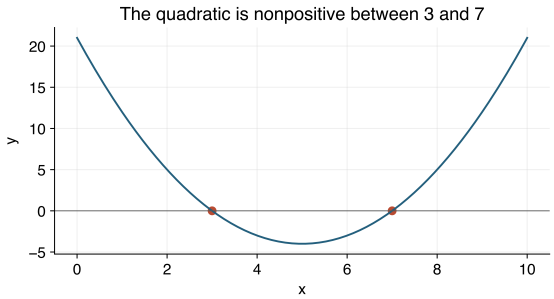


- The solution is the crossing point: x = 7, y = 5 satisfies both equations
- Match each part of the diagram to the text.

Inequalities

- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Figure: Inequalities

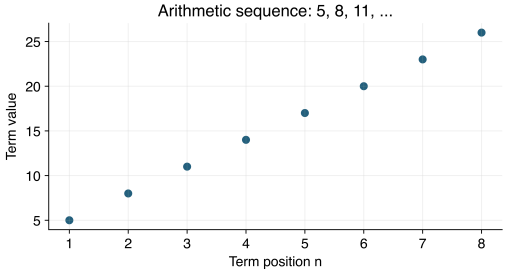


- $x^2 - 10x + 21 = (x - 3)(x - 7) \leq 0$ for $3 \leq x \leq 7$
- Read the axes and units first.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** 通项, gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** 等差数列 adds the same number each time.

Figure: Sequences



- Add 3 each time: **nth term** 3n + 2, so the eighth term is 26
- Match each part of the diagram to the text.

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.2 (Algebra).
- Muhammad ibn Musa al-Khwarizmi, *The Compendious Book on Calculation by Completion and Balancing* (Baghdad, about 820), the problem $x^2 + 10x = 39$.
- AQA, *GCSE Mathematics* Higher Papers 1 and 3 (June 2024), the linear-equation and **identity** questions.

Common errors

- Writing $(x+3)^2 = x^2 + 9$; it is $x^2 + 6x + 9$.
- Cancelling single terms in a fraction: $\frac{x+6}{x}$ is not 6.
- Doing f first in $fg(x)$; g is applied first.

Common errors (2)

- Forgetting to turn the inequality sign round after dividing by a negative number.
- Giving only one **root** of a quadratic, or one crossing point of a line and a curve.
- Rounding $\sqrt{57}$ before the final step.
- Writing the **nth term** of 5, 8, 11, ... as $n + 3$; the difference 3 must multiply n.

Key terms

- **Identity**: a statement that is true for every value of the letter, written with $\equiv$.
- **Gradient**: the steepness of a line, the change in y for each increase of 1 in x.
- **Root**: a value of x that makes an equation true; where a graph crosses the x-axis.

Key terms (2)

- **Discriminant**: $b^2 - 4ac$, which tells you how many roots a quadratic has.
- **Elimination**: removing one unknown by adding or subtracting equations.
- **Iteration**: repeating a formula, using each answer as the next input.
- **nth term**: the rule that gives any term of a sequence from its position.

Check yourself

- 1.
- Expand and simplify $(x+5)(x-2)$. 2.
- Factorise $x^2+7x+12$, and hence solve $x^2+7x+12=0$. 3.
- Make r the subject of $A=\pi r^2$. 4.