

Probability – native teaching supplement

AQA · GCSE · Mathematics

Supported teaching and tier boundary

8300: Probability. Version: Version 1.0, 12 September 2014; first examination 2017.

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

■ Probability trees and outcomes · Foundation

A probability lies between 0 and 1. Exhaustive, mutually exclusive outcomes have probabilities summing to 1. Multiply successive branch probabilities and add separate routes to an outcome.

$$P(RR) = P(R_1)P(R_2 | R_1)$$

A bag contains 3 red and 2 blue counters. With replacement, $P(\text{two red}) = 3/5 \times 3/5 = 9/25 = 0.36$. Without replacement, the red-red branch is $3/5 \times 2/4 = 0.3$. Label each branch before multiplying.

Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.

Use a frequency table or a simple tree before calculating. Formal conditional probability formulae are outside this Foundation/Core support lesson.

■ Probability, trees and conditional reasoning · Higher

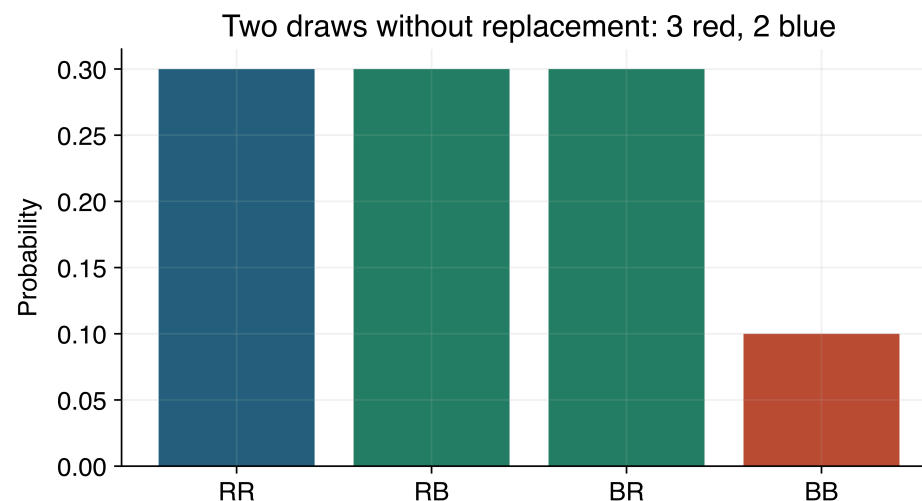
Multiply along a tree branch and add disjoint branches. With replacement, the composition stays fixed. Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.

Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability. Two disjoint events with positive probability are not independent.

A two-way table makes the restricted denominator visible. Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.



Original native-lesson illustration; labels belong to its worked example.

■ Experimental probability, fairness and expected outcomes · Foundation

Record each trial consistently and total the counts. Relative frequency estimates probability as outcome frequency/trials. For a stated probability p , expected count in n future trials is np ; it is a long-run average, not a guarantee. Unbiased trials with larger samples usually give more stable estimates, but cannot force exact agreement with theory. Probabilities lie from 0 to 1 and a mutually exclusive exhaustive list sums to 1.

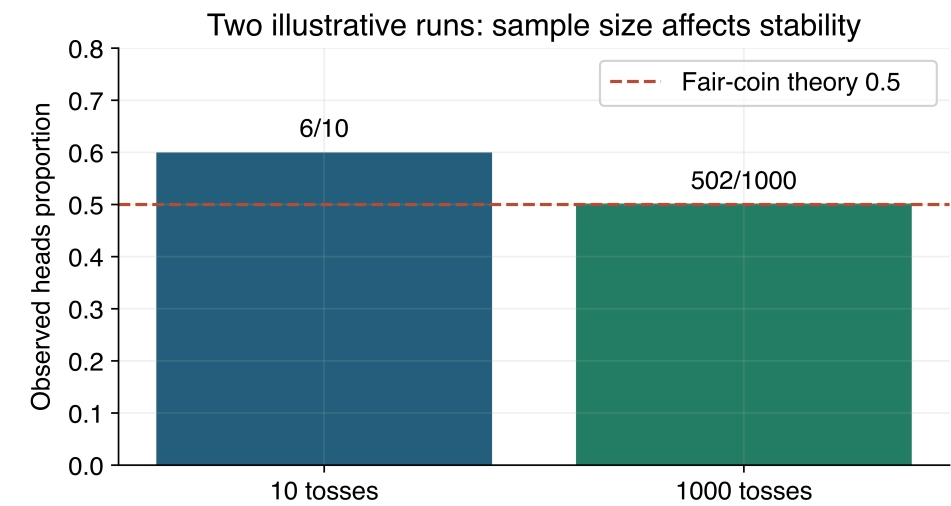
$$\hat{p} = \frac{f}{n}, \quad E = np$$

A spinner lands red 18 times in 60 spins, giving estimated $P(\text{red}) = 18/60 = 0.3 = 30\%$. Using this estimate predicts $0.3 \times 200 = 60$ red results in 200 future spins. If red, blue and green are exhaustive with probabilities 0.3, 0.45 and p , then $p = 1 - 0.75 = 0.25$. A fair coin has theoretical $P(\text{head}) = 0.5$; 100 tosses give expected heads 50, but 48 or 54 is possible. An experiment with 6 heads in 10 tosses estimates 0.6; one with 502 heads in 1000 estimates 0.502. These illustrative runs are not proof that error falls at every stage. Spin using the same method and record all results rather than stopping when a favourite outcome appears.

A larger biased sample can still be misleading. Expected does not mean certain. Mutually exclusive events cannot happen together; if categories overlap, do not simply add their probabilities as separate outcomes.

AQA P1–P5 uses frequency tables/trees, randomness/fairness, expected counts, the probability scale and empirical/theoretical comparison. State whether a value is ob-

served, estimated or theoretical.



Original native-lesson illustration; labels belong to its worked example.

■ Systematic possibilities and equally likely outcomes · Foundation

List outcomes systematically using a grid, table or tree. For equally likely outcomes, probability is favourable outcomes/total outcomes. Keep ordered outcomes distinct when the experiments have labelled first and second stages. A grid shows completeness and prevents duplicate counting; unequal probabilities need weights rather than a simple count.

$$P(E) = \frac{\text{favourable equally likely outcomes}}{\text{all equally likely outcomes}}$$

A coin and a fair die have 12 equally likely ordered outcomes: H1 to H6 and T1 to T6. Heads with an even die result has three outcomes, so probability is 3/12=1/4. Two dice have 36 ordered pairs. Total seven arises from (1,6),(2,5),(3,4),(4,3),(5,2),(6,1), so probability is 6/36=1/6. A double has six outcomes and probability 1/6. Total at least eleven occurs in (5,6),(6,5),(6,6), giving 1/12. With a spinner divided into unequal sectors, the named colours are not automatically equally likely; use the sector proportions or a justified experimental estimate.

Counting totals rather than equally likely dice pairs gives wrong weights. State whether order matters. A possibility table lists outcomes; its entries are not automatically equiprobable.

AQA P6/P7 uses tables, grids and trees to enumerate theoretical possibilities. Explain why the selected elementary outcomes have equal probability before dividing counts.

36 equally likely cells; six sums equal seven

6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

Original native-lesson illustration; labels belong to its worked example.

■ Venn diagrams, unions and two-way counts · Foundation

Place the overlap in a Venn diagram first, then fill the only-regions and neither-region. Union means at least one named set; intersection means both; complement means outside a named set in the stated universal group. Two-way tables classify every observation by one category from each of two variables. Check row, column and grand totals.

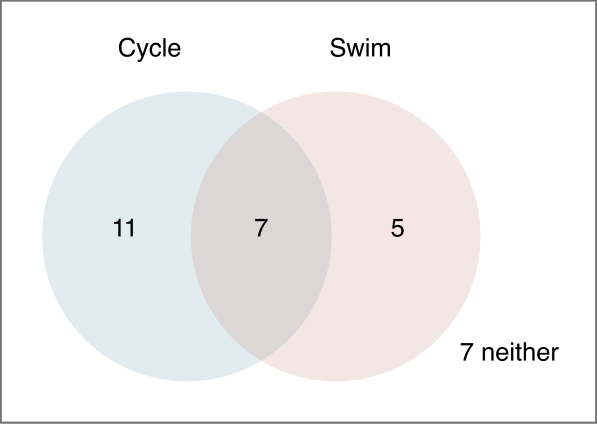
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

In a class of 30, 18 cycle, 12 swim and 7 do both. Cycling only is 11 and swimming only 5; at least one is 11+7+5=23, leaving 7 neither. A random student has probability 7/30 of both and 23/30 of at least one. The two-way table has cycle-and-swim 7, cycle-not-swim 11, not-cycle-swim 5 and neither 7. The cycle row totals 18, swim column totals 12 and grand total 30. The outcomes both, cycling only, swimming only and neither are mutually exclusive and exhaustive, so their probabilities sum to 1. A frequency tree starts at 30, branches to cycle 18 and not-cycle 12, then to swim/not-swim counts 7/11 and 5/7. Each pair of terminal counts sums back to its parent.

Do not add the overlap twice. Neither is outside both circles, not just outside their overlap. A universal group must be stated before taking a complement.

AQA P4/P6 uses exhaustive events, systematic sets and Venn/table representations. Translate the words both, either/at least one, only and neither into the correct counted regions.

30 students: overlap first, then only and neither



Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Original native-lesson illustration; labels belong to its worked example.

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■ Conditional probability with tables and expected frequencies · Higher

Conditioning restricts the denominator to the given group. In a table, divide the intersection count by the condition’s row/column total. $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$ when $P(B) > 0$. Reversing the condition usually changes the denominator. Expected-frequency trees help interpret percentages without treating the two conditions as interchangeable.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Using a class of 30 with 18 cyclists, 12 swimmers and 7 doing both, $P(\text{swim given cycle}) = 7/18$, while $P(\text{cycle given swim}) = 7/12$. The unconditional swim probability is $12/30 = 0.4$. For an expected cohort of 1000, suppose 20% have a condition; a test is positive for 90% with it and 10% without it. Expected positive counts are 180 from 200 with the condition and 80 from 800 without it. Of 260 positive tests, the conditional proportion with the condition is $180/260 = 9/13 \approx 0.6923$, not 90%. These are stated illustrative model rates, not claims about a real diagnostic test.

Given positive and positive given condition are different questions. Use the conditioned group total, not the grand total. A rare starting category can make false-positive counts significant even when the detection rate is high.

AQA P9 Higher requires conditional calculations and interpretation using two-way tables, trees and Venn diagrams. Name the restricted group and retain expected counts until the final ratio.

Original independent transfer

■ Foundation

A bag has three red and two blue counters. Two are drawn without replacement. Make a complete possibility tree and find the probability of different colours. Explain how the second-stage denominators change.

■ Foundation worked solution

First probabilities are $R:3/5$, $B:2/5$. After R, the second probabilities are $R:2/4$, $B:2/4$. After B they are $R:3/4$, $B:1/4$. Terminal probabilities RR, RB, BR, BB are $6/20, 6/20, 6/20, 2/20$ and sum to one. Different colours are the disjoint RB and BR paths, so probability is $6/20 + 6/20 = 3/5$. Multiplying stages follows a path; adding combines mutually exclusive paths. Replacement would change the model.

■ Higher

In 50 students, 30 study art, 25 study music and 15 study both. Find music given art and art given music. Are the events independent? Reconcile the counts using a two-way table.

■ Higher worked solution

Both=15, art only=15, music only=10 and neither=10; their sum is 50. $P(M \mid A) = 15/30 = 1/2$ while $P(A \mid M) = 15/25 = 3/5$. The intersection probability is $15/50 = 3/10$, equal to $(30/50)(25/50) = 3/10$, so these events are independent in this finite model. Their unequal conditional probabilities do not contradict independence, since independence compares each condition with the matching unconditional probability: $P(M) = 1/2$ and $P(A) = 3/5$.

Terms

conditional probability .

relative frequency .

sample space .

intersection .