

Ratio, proportion and rates of change – native teaching supplement

AQA · GCSE · Mathematics

Supported teaching and tier boundary

8300: Ratio, proportion and rates of change. Version: Version 1.0, 12 September 2014; first examination 2017.

Foundation teaching and Higher additions are labelled below. This reference packages the existing native-lesson crosswalk. It does not certify unreviewed specification rows or a whole qualification. Original diagnostics are separate and are not reproduced.

■ Percentages, ratio and proportional reasoning · Foundation

A p% increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$. Reverse a percentage by dividing by the multiplier. In a ratio, first find the total number of parts.

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

Let the original price be P. The model is sale price=0.8P. Hence $P=240/0.8=300$. A later 20% increase gives $240 \times 1.2=288$, so the two changes do not cancel.

A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.

Use percentage multipliers and divide a total into ratio parts. This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.

■ Percentages, ratio and proportional reasoning · Higher

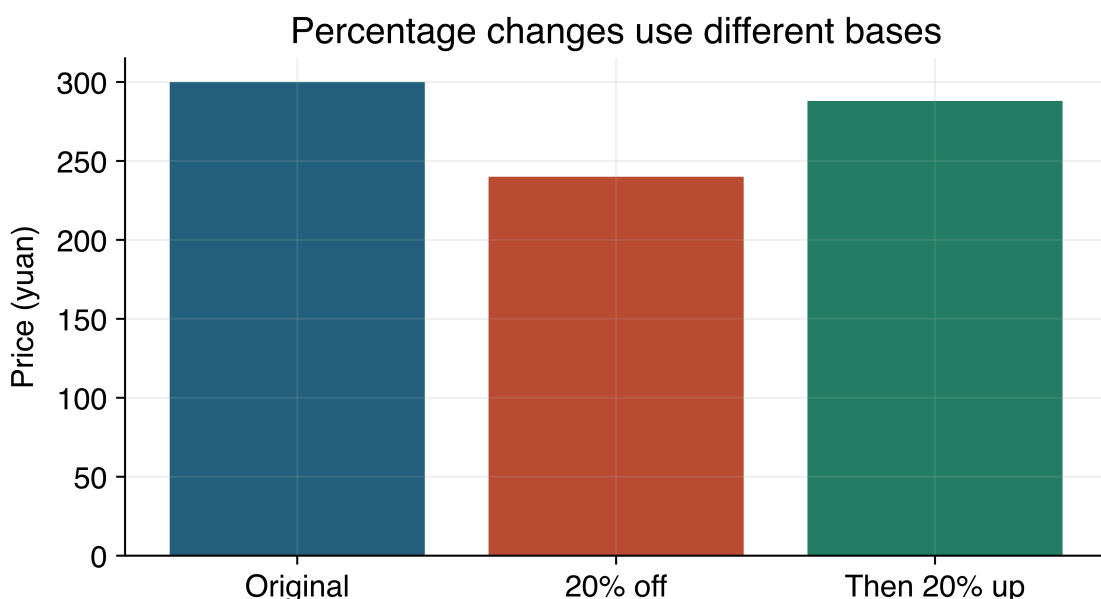
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A percentage uses a stated base. Subtracting the percentages loses that base. For compound change, multiply the multipliers; do not add the percentages.

For direct proportion use $y=kx$; for inverse proportion use $y=k/x$. Calculate k from a known pair before using a new value. State what you held constant.



Original native-lesson illustration; labels belong to its worked example.

■ Ratios, equivalent proportions and mixtures · Foundation

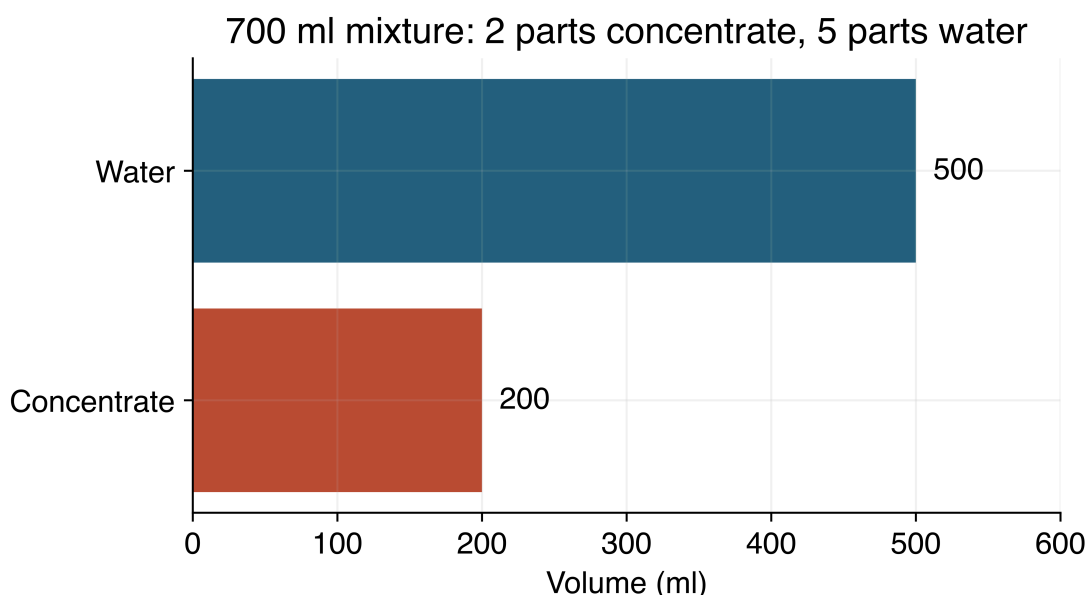
Put quantities in the same units before simplifying a ratio. Divide every part by the same nonzero factor. For $a:b$ the whole has $a+b$ parts; the first share is $a/(a+b)$ of the whole but a/b of the other share. Equivalent ratios have the same multiplicative relationship: $a/b=c/d$ gives $ad=bc$ when denominators are nonzero.

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc, \quad C = \frac{2}{7}V, \quad W = \frac{5}{2}C$$

For 700 ml at 2:5, each part is 100 ml: concentrate 200 ml and water 500 ml. Concentrate:whole=2:7, while concentrate/water=2/5. Water/concentrate=5/2, a fraction greater than 1. Doubling both gives 400:1000=2:5. If concentrate is x and water y , the recipe requires $y=(5/2)x$, a straight line through the origin. For 300 ml concentrate, water is 750 ml and the total is 1050 ml. A batch with 200 ml concentrate and 600 ml water has ratio 1:3 and is weaker. To simplify 1.5 litres:500 ml, first write 1500:500=3:1.

Adding the same amount to both shares generally changes the ratio. Do not divide by 2+7 when the stated ratio is already part:whole 2:7; the whole is seven parts. A concentration fraction uses total volume as denominator.

AQA R3–R8 connects ratio notation, fractions, equivalent proportions and linear functions. Check that both shares sum to the stated total, and name which quantity is compared with which. Use a ratio table to scale a mixture without assuming a fixed additive difference.



Original native-lesson illustration; labels belong to its worked example.

■ Scale drawings and maps · Foundation

In a scale 1:n, one drawing unit represents n of the same real unit. Multiply a drawing length by n to obtain the real length; divide a real length by n to draw it. Then convert the unit. Measure only when the diagram explicitly supplies an accurate scale.

$$L_{\text{real}} = nL_{\text{drawing}}$$

At 1:25,000, 4 cm represents 100,000 cm=1000 m=1 km. A real 1.5 km path is 150,000 cm, so it measures 150,000/25,000=6 cm on the map. A room 6 m by 4 m drawn at 1:100 becomes 6 cm by 4 cm because each metre is 100 cm. Its drawing diagonal is $\sqrt{(6^2+4^2)} \approx 7.21$ cm and the real diagonal is about 7.21 m. Enlarging the printed map changes its numerical scale: doubling drawing lengths halves the scale denominator. A scale bar printed with the map enlarges with it.

A ratio compares matching units. Never measure a diagram marked not to scale. A photocopied numerical scale can become invalid even though its scale bar still works. AQA R2 includes maps, scale factors and geometric problems. Label drawing and real dimensions separately; reverse the calculation to verify the scale. Use an exact ratio until the context asks for rounding.

Match units before applying the scale

4 cm on map



1 km on ground at 1:25,000

Original native-lesson illustration; labels belong to its worked example.

■ Direct and inverse proportional relationships · Foundation

Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin. Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded. State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion. Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. The inverse model assumes equally productive workers on one fixed job.

An increasing graph need not be direct proportion. An inverse relationship is proportional to $1/x$, not to $-x$. Keep a product constant for inverse proportion and a quotient constant for direct proportion.

AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion. Interpret the units of k and the gradient. In Foundation use the supplied equations; Higher also constructs and interprets them from the context.

■ Direct and inverse proportional relationships · Higher

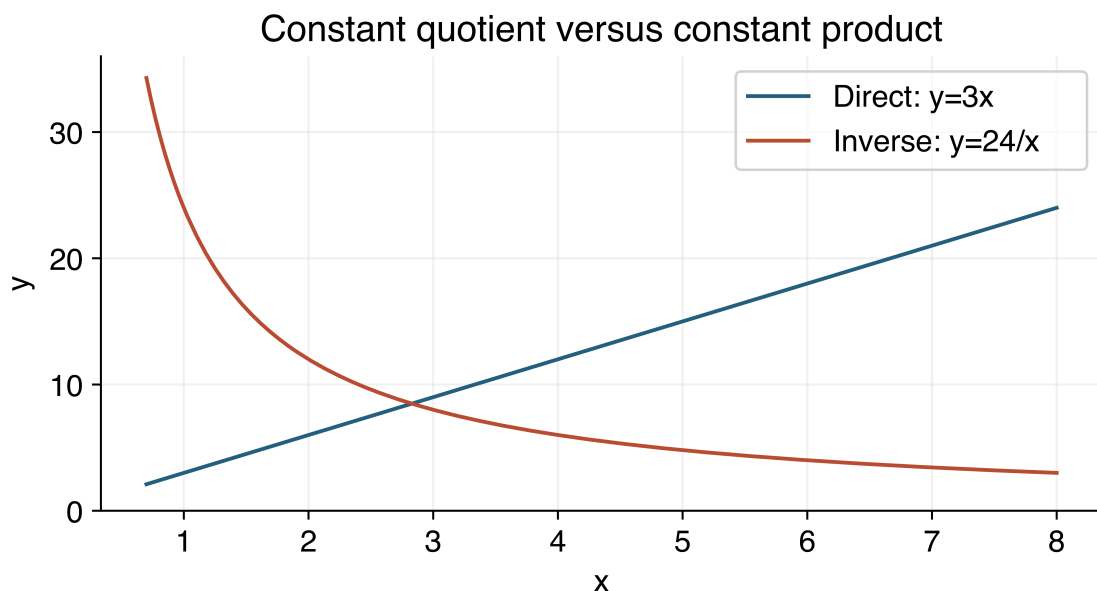
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■ Rates, unit prices, density and pressure · Foundation

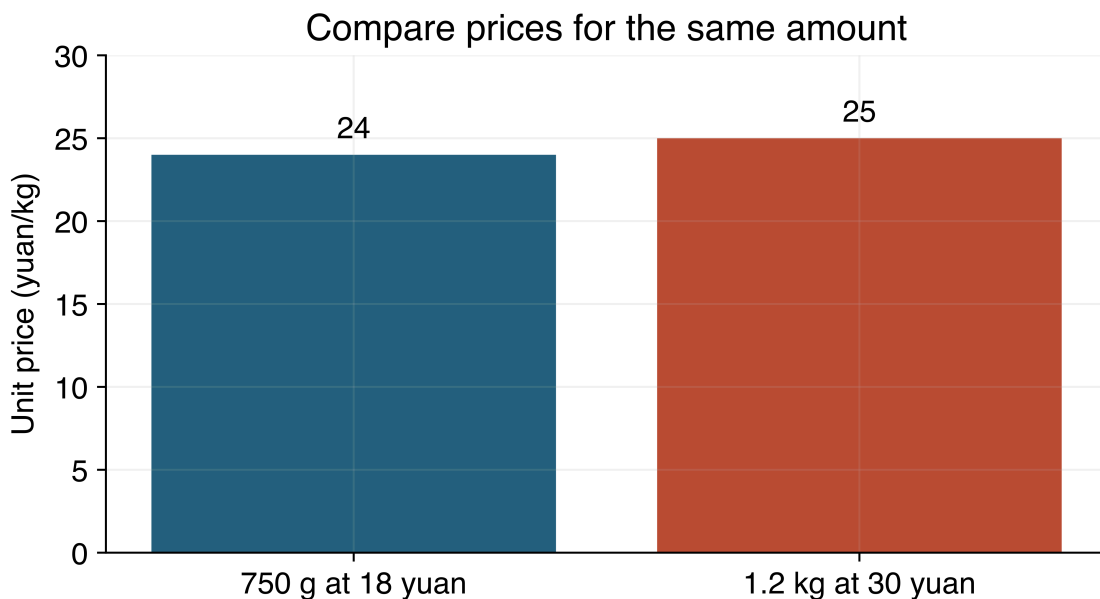
A compound rate divides one quantity by another: speed= d/t , pay=earnings/time, unit price=cost/amount, density=mass/volume and pressure=force/area. Rearrange these equations before substituting. Convert each dimension separately; converting cm^2 to m^2 uses a squared length factor. Comparisons must use the same units and conditions.

$$\rho = \frac{m}{V}, \quad p = \frac{F}{A}, \quad m = \rho V$$

A 750 g pack costing 18 yuan has unit price $18/0.75=24$ yuan/kg. A 1.2 kg pack at 30 yuan costs 25 yuan/kg, so the first is better value if quality and waste are equal. A block of mass 540 g and volume 200 cm^3 has density 2.7 g/cm^3 ; a 50 cm^3 piece of that material has mass 135 g. Since $1 \text{ g}=0.001 \text{ kg}$ and $1 \text{ cm}^3=0.000001 \text{ m}^3$, $2.7 \text{ g/cm}^3=2700 \text{ kg/m}^3$. A 120 N force over 0.03 m^2 gives pressure $4000 \text{ N/m}^2=4000 \text{ Pa}$. At fixed force, halving the area doubles pressure. An hourly pay rate of 48 yuan/hour gives 120 yuan for 2.5 hours.

Volume conversions cube the length factor and area conversions square it. A density of 2.7 g/cm^3 is not 2.7 kg/m^3 . A cheaper package may cost more per kilogram; include only comparable products.

AQA R1/R11 includes speed, pay, pricing, density and pressure in numerical and algebraic contexts. Show the rearrangement with named quantities and carry units through the answer; use an inverse check such as density $\times$ volume=mass.



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■ Length, area and volume scale factors · Foundation

For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 . Name the direction of the comparison: model to real or real to model. Recover k from an area ratio by a square root and from a volume ratio by a cube root. A change in only one dimension does not create similar solids. Higher links corresponding sides to equal trigonometric ratios.

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

Enlarging a box from $2 \times 3 \times 4$ to $6 \times 9 \times 12$ multiplies lengths by 3. Its volume changes from 24 to 648, a factor of 27; a 2×3 face changes area from 6 to 54, a factor of 9. Length ratio 2:5 corresponds to area ratio 4:25 and volume ratio 8:125. If similar shapes have areas 20 and 80 cm^2 , the larger-to-smaller length factor is $\sqrt{(80/20)}=2$. If similar solids have volumes 16 and 128 cm^3 , $k=\sqrt[3]{8}=2$.

Areas do not scale by k , and volumes do not scale by k^2 . Similarity needs every corresponding length to share the same factor. Reversing a ratio requires the reciprocal factor.

AQA R12 Foundation includes ratios and scale factors for lengths, areas and volumes. Higher adds links to similarity including trigonometric ratios. Check dimensions and compare a simple box or rectangle before applying the general factor.

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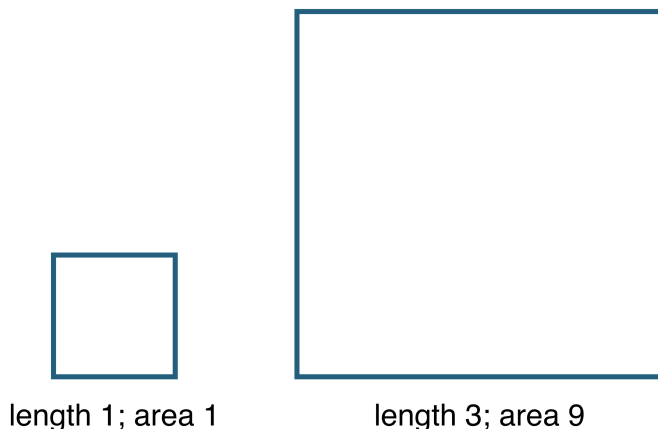
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Higher adds links to similarity including trigonometric ratios. Check dimensions and compare a simple box or rectangle before applying the general factor.

A length factor of 3 acts in both area dimensions



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■ Simple interest, compound growth and decay · Foundation

A percentage r corresponds to decimal $r/100$; increases use multiplier $1+r/100$ and decreases $1-r/100$. Simple interest on principal P for n periods at rate r is $Prn/100$, giving balance $P(1+rn/100)$. Compound balance is $P(1+r/100)^n$. Decay uses the corresponding decreasing multiplier. Use matching rate and period units, and divide by the multiplier to recover an original value.

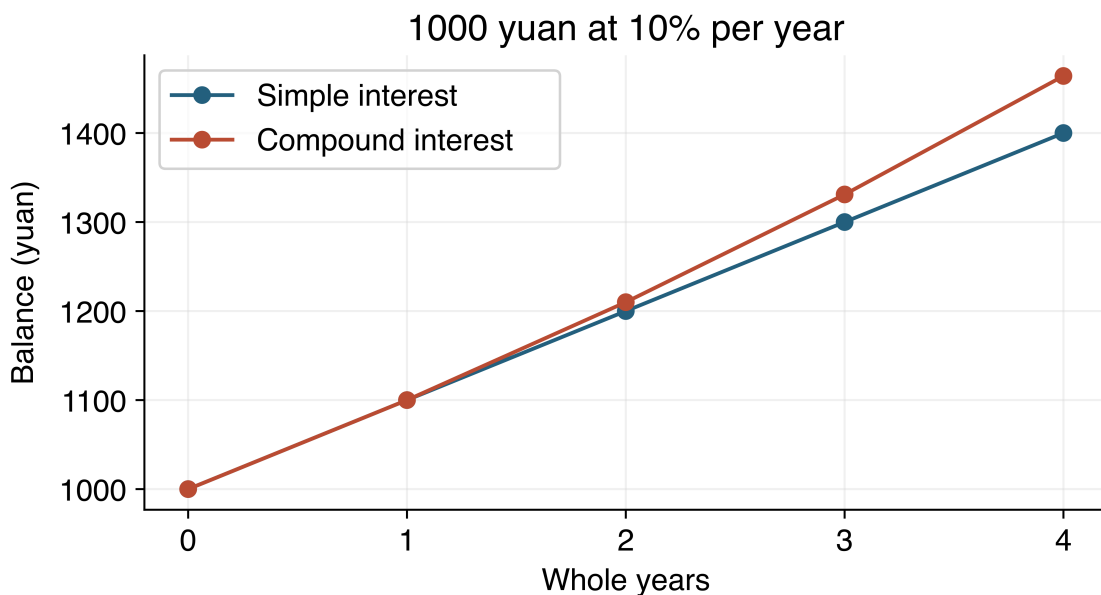
$$B_n = P \left(1 + \frac{r}{100} \right)^n$$

At 10% per year, 1000 yuan earns simple interest 100 per year: after two years interest is 200 and balance is 1200. Compound balances are 1100 after year one and 1210 after year two. A machine worth 800 yuan losing 20% each year becomes 640 then 512, not 480: the second loss is 20% of 640. An 800-yuan sale price after a 20% reduction corresponds to original price $800/0.8=1000$. Growth from 50 to 65 is $(65-50)/50 \times 100 = 30\%$; 65 is 130% of 50. If a compound balance must first exceed 1300 at 10%, year two gives 1210 and year three 1331, so three whole years are needed.

Use the original value as the denominator for percentage change. Repeated 20% decreases do not subtract 40% of the initial amount. A rate per year cannot be treated as a rate per month without a specified conversion.

AQA R9/R16 includes percentage comparisons, original values, simple interest and

repeated growth/decay. State whether the task asks for interest alone or total balance, and interpret whole-period threshold answers.



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■ Iterative processes with repeated deposits · Higher

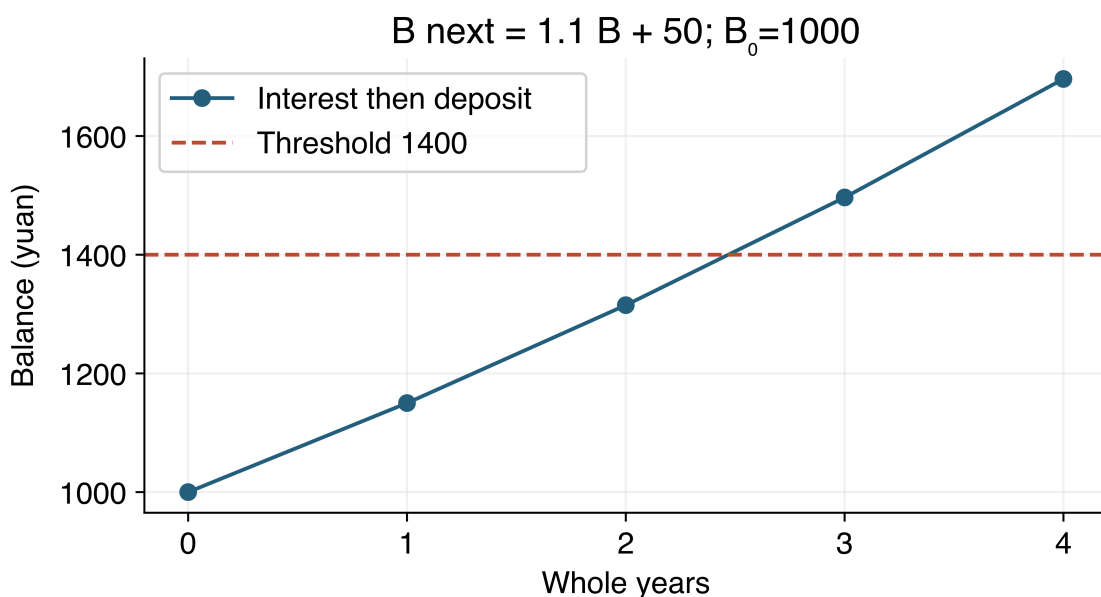
Write an update rule and an initial value. Apply the operations in their stated order, using the previous output as the next input. A recurrence $B_{\text{next}} = 1.1B + 50$ differs from $1.1(B + 50)$. Tables can locate a first whole-period threshold; verify both the preceding and crossing values. Iteration can model growth, decay or other repeated processes, not just solve equations.

$$B_{n+1} = 1.1B_n + 50, \quad B_0 = 1000$$

With $B_0 = 1000$ and $B_{n+1} = 1.1B_n + 50$, $B_1 = 1150$, $B_2 = 1315$ and $B_3 = 1496.5$. The balance first exceeds 1400 after three years because $1315 \leq 1400 < 1496.5$. If the deposit preceded interest, $B_1 = 1.1 \times 1050 = 1155$, five yuan greater. A decay-and-top-up rule $V_{\text{next}} = 0.8V + 20$, starting at 200, gives 180 then 164. A fixed point satisfies $V = 0.8V + 20$, hence $V = 100$; values above 100 decrease toward it. A fixed point is a value preserved by the update, not a claim that every finite step reaches it exactly.

Preserve operation order and use the updated value each time. Do not round early or confuse the initial value with the first updated value. A continuous fractional-period estimate does not answer a whole-period question by itself.

AQA R16 Higher extends growth/decay to general iterative processes. State the model assumptions, initial value, recurrence and threshold interpretation. Use substitution to check a proposed fixed point without calculus.



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Original independent transfer

■ Foundation

A 1:25000 map shows a road as 6.4 cm. Find its real length in kilometres. A cyclist travels it in eight minutes. Find average speed in kilometres per hour, showing the unit conversion.

■ Foundation worked solution

Real length $L = kl = 25000(6.4 \text{ cm}) = 160000 \text{ cm} = 1.6 \text{ km}$. Time $t = 8/60 \text{ h} = 2/15 \text{ h}$. Average speed $v = L/t = (1.6 \text{ km})/(2/15 \text{ h}) = 12 \text{ km/h}$. The map ratio compares like units before conversion. The average does not assert constant instantaneous speed.

■ Higher

A savings account starts at 1000 CNY. Each year it receives 5% interest, then a 100 CNY deposit. Write a recurrence and find the balance after two years. Compare with depositing before interest each year, explaining the difference.

■ Higher worked solution

Interest then deposit gives $B_{n+1} = 1.05B_n + 100$, with $B_0 = 1000$. Thus $B_1 = 1150$ and $B_2 = 1307.50$ CNY. Deposit first gives $C_{n+1} = 1.05(C_n + 100)$, so $C_1 = 1155$ and $C_2 = 1317.75$ CNY. The difference is 10.25 CNY: the first early deposit earns five CNY an extra year and that advantage earns 5% in year two, while the second early

deposit adds another five CNY. Operation order is part of the model.

Terms

- multiplier .
- part-to-whole ratio .
- scale .
- constant of proportionality .
- density .
- area scale factor .
- compound interest .
- recurrence relation .