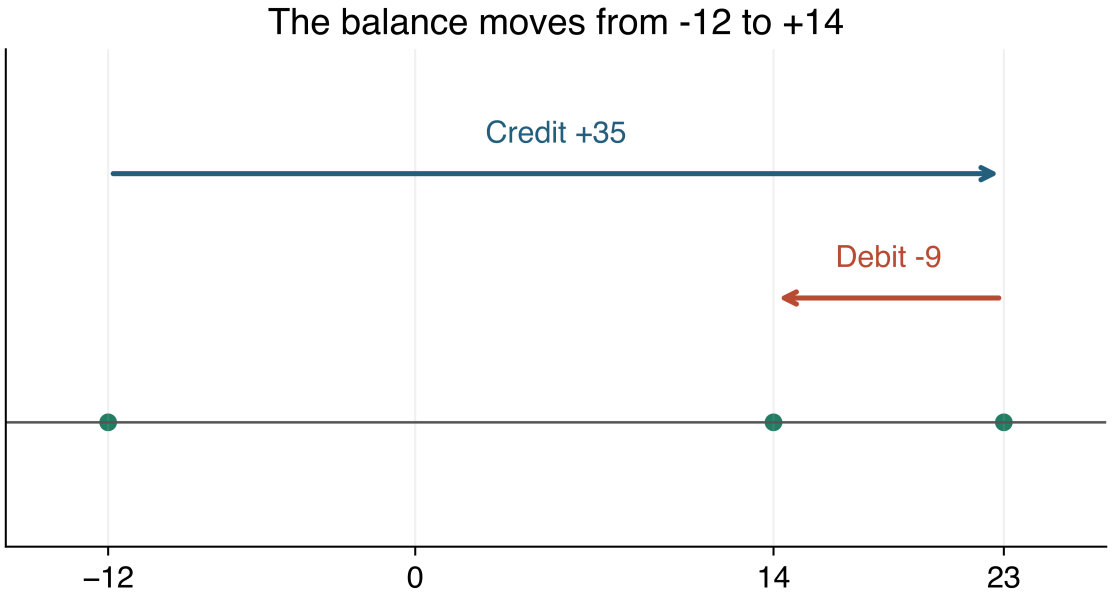
Number – worked practice

AQA · GCSE · Mathematics

Foundation practice: signs, fractions, decimals and units

This supplement gives more worked practice for the Number topic. Each example shows the method an examiner expects to see. Try each one yourself before you read the solution.

Worked example 1: negative numbers. At 6 a.m. the temperature in Harbin is -12 °C. By 2 p.m. it has risen by 35 degrees, and by midnight it has fallen by 9 degrees. Find the temperature at midnight. Start at -12 and move 35 to the right on the number line: $-12 + 35 = 23$. Then move 9 to the left: $23 - 9 = 14$. The temperature at midnight is 14 °C.

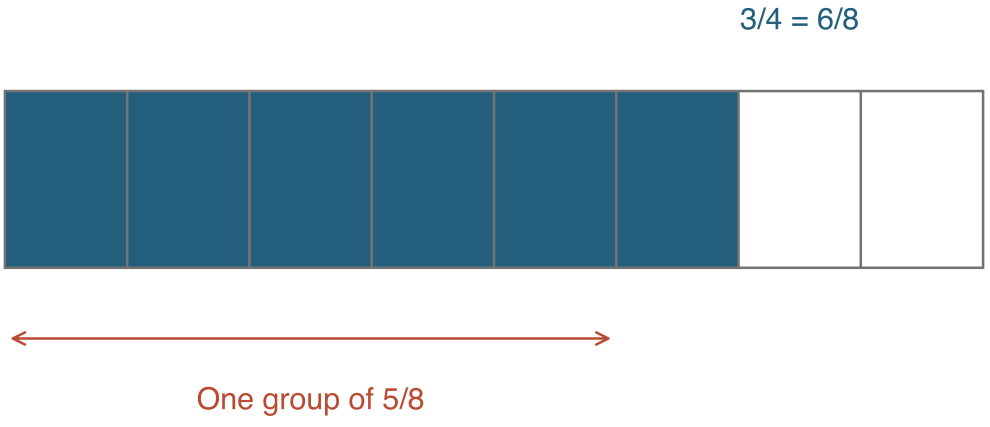


−12 + 35 − 9 = 14

Worked example 2: dividing fractions. How many bottles of $\frac{5}{8}$ litre can be filled from $\frac{3}{4}$ litre of juice? Write both with the same denominator: $\frac{3}{4} = \frac{6}{8}$. Then $\frac{6}{8} \div \frac{5}{8} = \frac{6}{5}$. You can also multiply by the **reciprocal** 倒数 (the fraction turned upside down): $\frac{3}{4} \times \frac{8}{5} = \frac{24}{20} = \frac{6}{5}$. The answer is an **improper fraction** 假分数, a fraction whose top is bigger than its bottom: $\frac{6}{5} = 1\frac{1}{5}$. So one bottle is filled completely, and a fifth of a second bottle.

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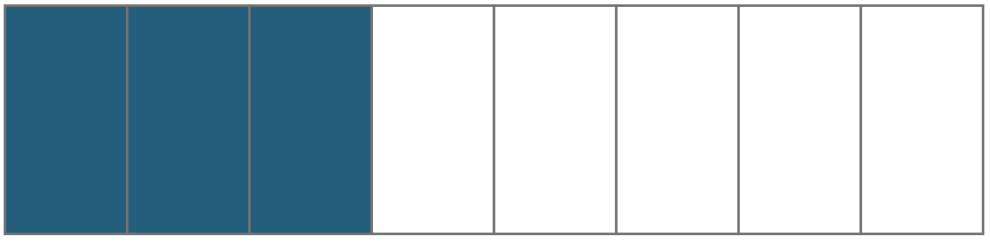
Six eighths contain 6/5 groups of five eighths



3/4 ÷ 5/8 = 6/8 ÷ 5/8 = 6/5: one group of five eighths and one fifth of another

Terminating and recurring decimals. A **terminating decimal** 有限小数 stops, such as $\frac{3}{8} = 0.375$. A fraction in its simplest form gives a terminating decimal only when its denominator has no prime factors except 2 and 5: $8 = 2^3$, so $\frac{3}{8}$ terminates, but $\frac{1}{3} = 0.333\ldots$ does not.

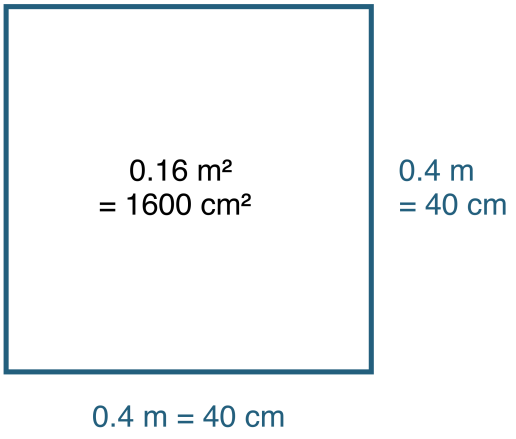
Different exact forms, the same shaded fraction



3/8 = 0.375 = 375/1000: three forms of the same shaded fraction

Worked example 3: area units. A floor tile is 0.4 m by 0.4 m. Find its area in cm^2. Convert the lengths first: 0.4 m = 40 cm, so the area is $40 \times 40 = 1600$ cm^2. Working in metres gives 0.16 m^2, and the **conversion factor** 换算因子 for area is $100^2 = 10\,000$, so $0.16 \times 10\,000 = 1600$ cm^2 again. Multiplying 0.16 by 100 is the common mistake.

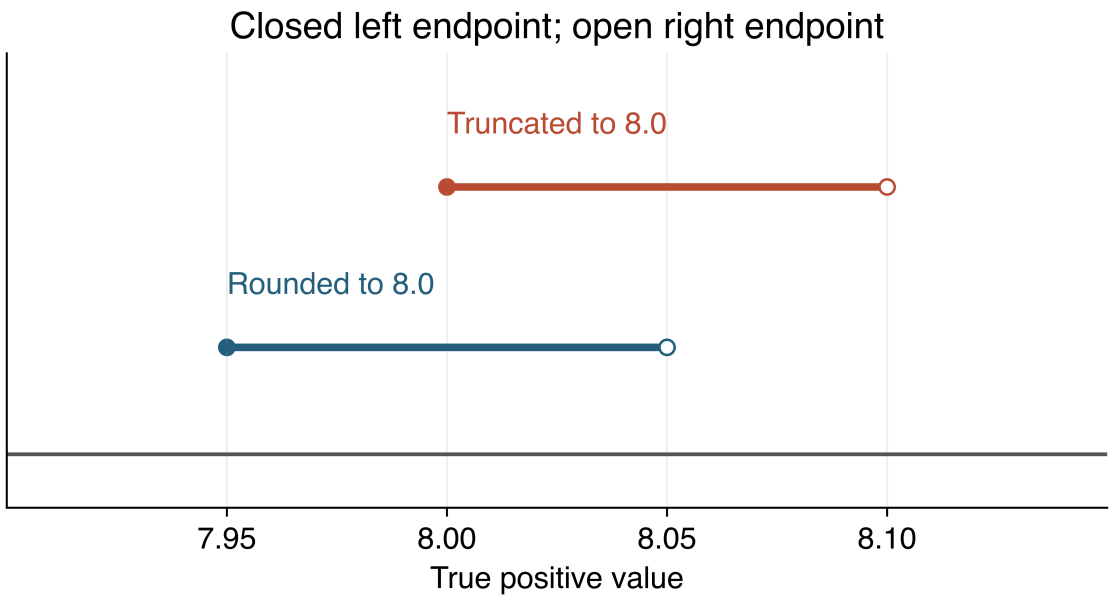
Convert both dimensions before multiplying



Convert first: 0.4 m = 40 cm, so 0.16 m^2 = 40 × 40 = 1600 cm^2, not 16 cm^2

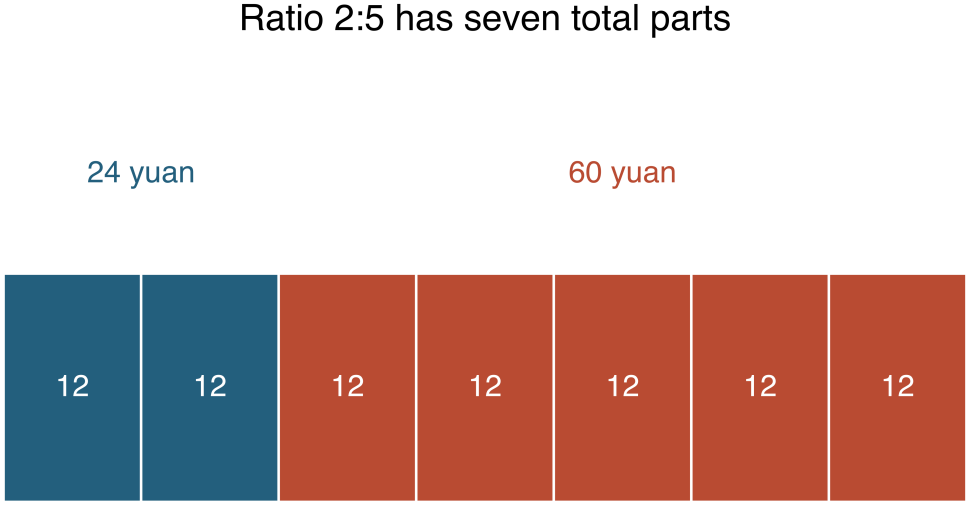
Significant figures. The first **significant figure** 有效数字 is the first digit that is not zero. To 2 significant figures, 0.004 076 is 0.0041 and 38 512 is 39 000. A length of 8.0 cm given to 1 decimal place could be from 7.95 cm up to 8.05 cm; if the 8.0 was truncated, it could be from 8.00 cm up to 8.10 cm.

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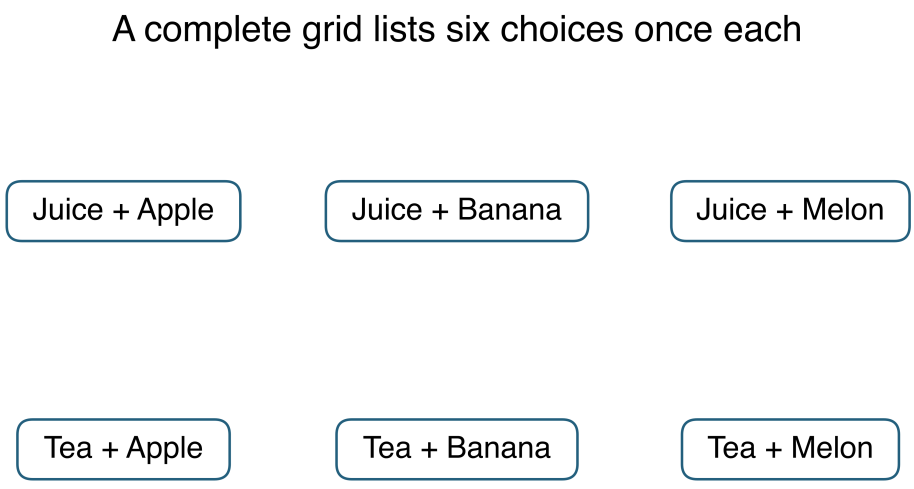
Rounded to 8.0: $7.95 \le x < 8.05$; truncated to 8.0: $8.00 \le x < 8.10$

Worked example 4: sharing in a ratio. Two teams share a prize of 84 yuan in the **ratio** 比 $2 : 5$. There are $2 + 5 = 7$ parts, so one part is $84 \div 7 = 12$ yuan. The teams get $2 \times 12 = 24$ yuan and $5 \times 12 = 60$ yuan. Check: $24 + 60 = 84$. The first team's share is $\frac{2}{7}$ of the prize, not $\frac{2}{5}$.



Ratio $2 : 5$ of 84 yuan: one part is $84 \div 7 = 12$, so 24 and 60 yuan

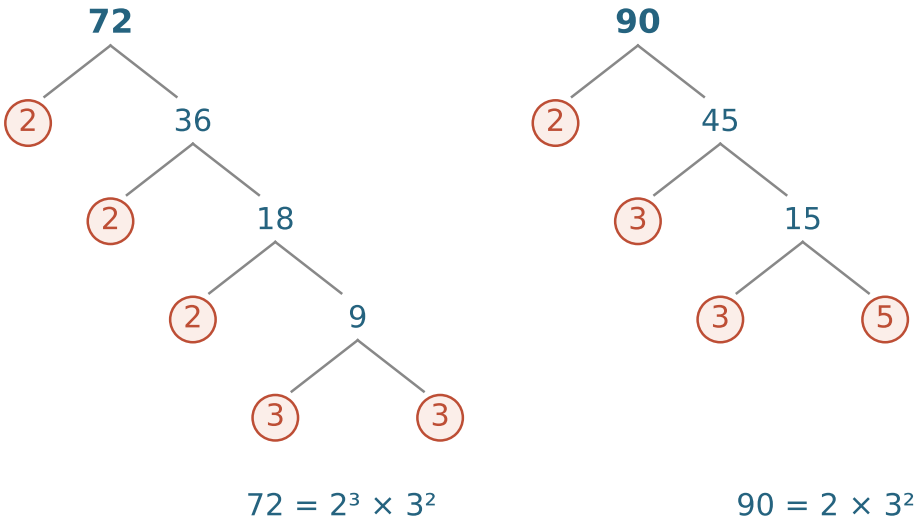
Worked example 5: listing outcomes. A lunch deal is one drink (juice or tea) and one fruit (apple, banana or melon). A **systematic list** 系统列表 keeps the first choice fixed while the second changes: juice-apple, juice-banana, juice-melon, tea-apple, tea-banana, tea-melon. That is 6 combinations, which matches the product rule: $2 \times 3 = 6$.



2 drinks × 3 fruits = 6 combinations, each listed once

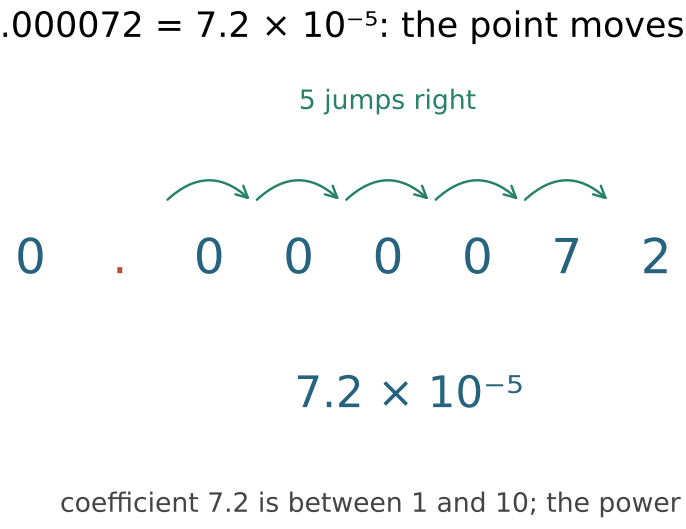
Higher practice: factors, indices, surds and bounds

Worked example 6: factor trees. Split 72 and 90 into primes with factor trees. Each **prime factor** 质因数 is a branch that cannot split further. $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. HCF $= 2 \times 3^2 = 18$; LCM $= 2^3 \times 3^2 \times 5 = 360$. So 72 pencils and 90 pens can make at most 18 identical bags, each with 4 pencils and 5 pens.



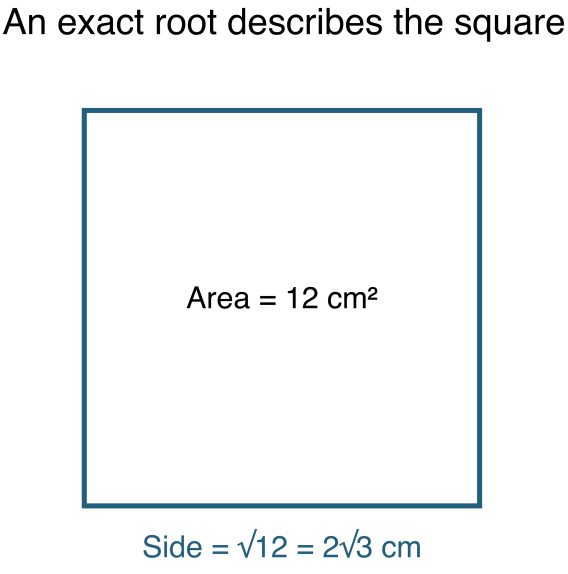
Factor trees: $72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$, so HCF $= 2 \times 3^2 = 18$

Worked example 7: indices and standard form. The **index** 指数 is the small raised number in a power. To write 0.000 072 in standard form, move the decimal point 5 places to the right to get 7.2, so $0.000\,072 = 7.2 \times 10^{-5}$. With fractional indices, take the root first: $16^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$. Remember that index laws work only for multiplying and dividing: $2^3 + 2^4 = 8 + 16 = 24$, not 2^7.



Moving the point 5 places right gives 7.2, so $0.000072 = 7.2 \times 10^{-5}$

Worked example 8: surds. A square has an area of 12 cm^2. Its side is $\sqrt{12}$ cm, an exact **surd** 根式. Simplify: $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$ cm. In the same way $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$. Brackets with surds expand like algebra: $(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = 1$.

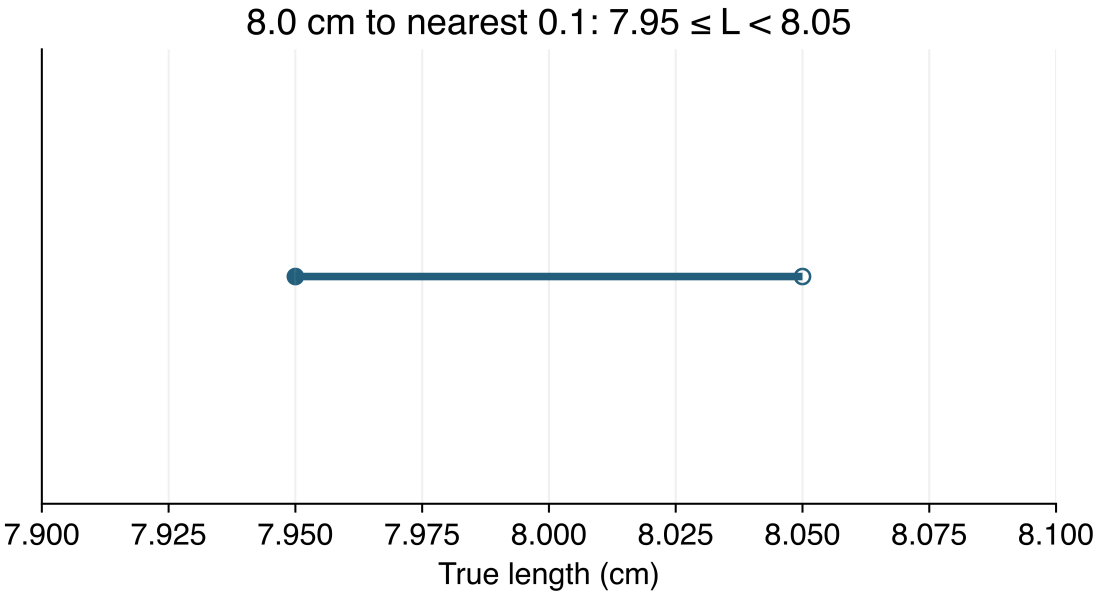


Side = √12 = √(4 × 3) = 2√3 cm

Recurring decimals. A **recurring decimal** 循环小数 such as $0.1\dot{6} = 0.1666\ldots$ has a part that does not repeat. Let $x = 0.1666\ldots$. Then $10x = 1.666\ldots$ and $100x = 16.666\ldots$. Subtract: $90x = 15$, so $x = \frac{15}{90} = \frac{1}{6}$.

Worked example 9: bounds. A rectangle is 8.0 cm by 3.0 cm, each to the nearest 0.1 cm. The length is $7.95 \le l < 8.05$ and the width is $2.95 \le w < 3.05$. The **lower bound** 下界 of the area is $7.95 \times 2.95 = 23.4525$ cm^2 and the upper bound is $8.05 \times 3.05 = 24.5525$ cm^2. For length $\div$ width, the smallest value is $7.95 \div 3.05 = 2.61$ and the largest is $8.05 \div 2.95 = 2.73$ (3 s.f.): to make a fraction small, make the top small and the bottom big.

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8.0 cm to the nearest 0.1 cm: $7.95 \le L < 8.05$

Estimating a root. Without a calculator, $4^2 = 16 < 20 < 25 = 5^2$, so $\sqrt{20}$ is between 4 and 5. Testing $4.5^2 = 20.25$ shows that $\sqrt{20}$ is just below 4.5.

Exam-style problems

Problem 1 (Foundation). A shop has 84 pencils and 126 pens. It makes identical packs and uses every pencil and every pen. Find the greatest number of packs and what each pack contains.

Solution. $84 = 2^2 \times 3 \times 7$ and $126 = 2 \times 3^2 \times 7$, so the HCF is $2 \times 3 \times 7 = 42$. There are 42 packs, each with $84 \div 42 = 2$ pencils and $126 \div 42 = 3$ pens.

Problem 2 (Foundation). A delivery charge of 18.60 yuan is shared equally by six friends. Then each friend pays 15% more because of a late fee. How much does each friend pay in the end?

Solution. $18.60 \div 6 = 3.10$ yuan. Adding 15% means multiplying by 1.15: $3.10 \times 1.15 = 3.565$, which is 3.57 yuan to the nearest fen.

Problem 3 (Higher). Show that $\frac{\sqrt{48}+\sqrt{12}}{\sqrt{3}}$ is a whole number.

Solution. $\sqrt{48} = 4\sqrt{3}$ and $\sqrt{12} = 2\sqrt{3}$, so the top is $6\sqrt{3}$, and $\frac{6\sqrt{3}}{\sqrt{3}} = 6$.

Problem 4 (Higher). A runner covers 100 m, measured to the nearest metre, in 12.4 s, measured to the nearest 0.1 s. Find the upper bound of the runner's average speed.

Solution. The greatest speed uses the greatest distance and the shortest time: $100.5 \div 12.35 = 8.14$ m/s (3 s.f.).

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Terms

- **Prime factor**: a factor of a number that is also a prime number.
- **Index**: the power in a^n; it tells you how many copies of a are multiplied.
- **Lower bound**: the smallest value a rounded measurement could have had.
- **Reciprocal**: 1 divided by a number; the reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$.
- **Improper fraction**: a fraction whose numerator is bigger than its denominator.
- **Systematic list**: a list made in a fixed order so that no outcome is missed or repeated.
- **Surd**: an exact root that is not a whole number or fraction, such as $\sqrt{3}$.
- **Terminating decimal**: a decimal that stops.
- **Recurring decimal**: a decimal with digits that repeat for ever.
- **Conversion factor**: the number you multiply by to change units.
- **Significant figure**: a digit that counts for accuracy, starting from the first non-zero digit.
- **Ratio**: a comparison of the sizes of two or more parts of a whole.

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