

4.3 Pythagoras, trigonometry and vectors

Name: _____ Class: _____ Date: _____

Total: 28 marks**KEY WORDS** hypotenuse • exact value • collinear

Pythagoras, trigonometry and vectors

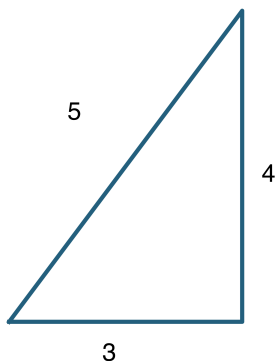
Objective

Apply Pythagoras' theorem and the trigonometric ratios in right-angled triangles in 2D and 3D, use exact trig values, apply the sine and cosine rules and $\frac{1}{2}ab \sin C$, and describe translations as vectors (rows G6, G20–G24).

1 Key knowledge and worked examples

Triangles and vectors. $a^2 + b^2 = c^2$; $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$; exact values from the $1, \sqrt{3}, 2$ and $1, 1, \sqrt{2}$ triangles. Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B}$; cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$; area $\frac{1}{2}ab \sin C$. A translation by column vector $\begin{pmatrix} x \\ y \end{pmatrix}$ adds x and y to the coordinates; its length is $\sqrt{x^2 + y^2}$.

Label the known sides before choosing a rule



Original native-lesson illustration; labels belong to its worked example.

2 Practice

2.1 A ladder footed 3 m from a wall reaches 4 m up it. Find the ladder's length. [3]

2.2 In a right triangle the opposite and adjacent sides of angle θ are 4 and 3. Find θ to 1 decimal place. [3]

2.3 Write down the exact value of $\sin 30^\circ + \cos 60^\circ$. [2]

2.4 A right triangle has hypotenuse 10 and an angle of 30° . Write down the exact lengths of the two other sides. [3]

2.5 (Higher.) Two sides of a triangle, 6 and 8, enclose 60° . Find the third side in surd form. [3]

2.6 (Higher.) In a triangle, $a = 4$ is opposite $A = 30^\circ$ and $B = 45^\circ$. Find b in surd form. [3]

3 Exam-style questions

3.1 (*June 2024 2H Q18, adapted.*) In a right triangle the hypotenuse is 23.7 and x is the side adjacent to the marked angle 52° . Work out x to 1 decimal place. [3]

3.2 (*June 2024 1H Q25, adapted.*) Show that the value of $6 \sin 30^\circ + 2 \cos 30^\circ \times 4 \tan 30^\circ$ is an integer. [3]

3.3 (*Original, Higher 3D.*) A cuboid has width 3 cm, depth 4 cm and height 12 cm. Work out the length of its space diagonal. [4]

3.4 (*Original.*) A translation maps $A(1, 2)$ onto $B(5, 5)$. Write the translation as a column vector and find its length. [2]
