

2.1.5 Algebraic Fractions

Name: _____ Class: _____ Date: _____

KEY WORDS excluded value

<!-- Session 2 native-skill companion -->

Objective

Practise algebraic fractions within the labelled native lessons. Public original tasks; locally authored answer guidance.

1 Key knowledge and worked examples

■ Algebraic fractions and excluded values · Higher

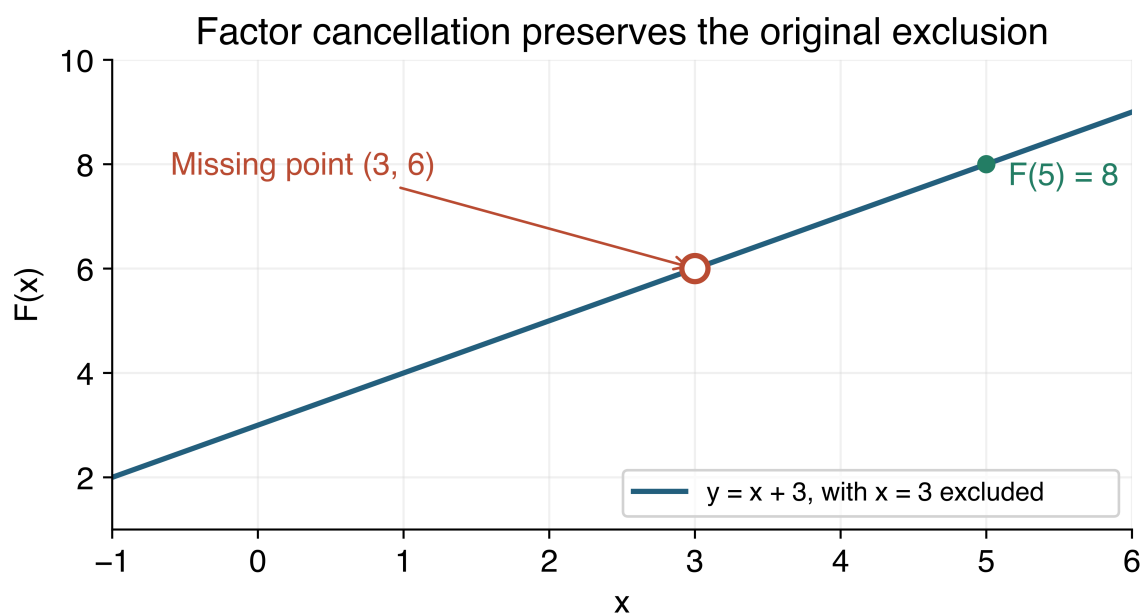
Factor every numerator and denominator before cancelling a common factor. A term joined by addition is not a cancellable factor. Record exclusions from the original expression first. For addition or subtraction use a common denominator, retaining brackets around the entire numerator. Multiply factored numerators and denominators. To divide, multiply by the reciprocal and exclude inputs making the divisor zero as well as inputs making any original fraction undefined.

$$\frac{x^2 - 9}{x - 3} = x + 3 \quad (x \neq 3)$$

For $F(x) = (x^2 - 9)/(x - 3)$, factor $x^2 - 9 = (x - 3)(x + 3)$. Thus $F(x) = x + 3$ for $x \neq 3$; $F(5) = 8$, but $F(3)$ is undefined, not 6. For addition, $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$, with $x \neq \pm 1$. At $x = 3$, both forms give 1. For subtraction, the numerator is $(x + 1) - 2(x - 1) = 3 - x$, so the minus sign acts on both terms. Multiplication $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$ gives $(x - 1)/(x + 1)$, but the original excludes $x = -2$ and $x = -1$. Division $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$ gives 1, yet $x = -2, -1, 1$ are excluded: $x = 1$ makes the divisor zero. Finally $F(x) = 6$ reduces to $x + 3 = 6$. Its only candidate $x = 3$ is forbidden, so this equation has no solution. Check candidates in the original equation before reporting them.

Never cancel the x in $(x + 2)/x$. A simplified denominator does not show all original restrictions. In a division task, check when the divisor is zero. Cross-multiplication can produce an excluded candidate; a formal root is not automatically a solution.

AQA A4 Higher includes algebraic fractions and valid cancellation. The equation example checks whether a candidate is inside the original domain. Explain each factor cancellation, retain original restrictions and check every equation candidate. These operations extend the existing manipulation example into a complete worked arithmetic sequence.



2 Practice

2.1 (Higher) For $F(x) = \frac{(x^2-9)}{(x-3)}$, find $F(5)$.

2.2 (Higher) Find $\frac{1}{(x-1)} + \frac{2}{(x+1)}$ at $x=3$.

2.3 (Higher) How many allowed solutions does $\frac{(x^2-9)}{(x-3)} = 6$ have?

3 Exam-style questions

3.1 Decide whether this claim is true. Justify with a concrete calculation or counterexample: Cancelling a denominator factor always makes its excluded input valid.
